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Rigidity of Closed Convex Hypersurfaces in Multidimensional Spaces of Constant Curvature

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In 1972, E.P. Sen'kin generalized the celebrated theorem of A.V. Pogorelov on the unique determination of closed convex surfaces by their intrinsic metrics in the Euclidean three-dimensional space E^3 to higher dimensional Euclidean spaces E^{n+1} under a mild assumption on the smoothness of hypersurfaces. In this paper, we remove that assumption and thereby establish a rigidity result for arbitrary closed convex hypersurfaces in E^{n+1}, $n \geq 3$. We also prove similar results in other model spaces of constant curvature.

Key words: rigidity, convex hypersurface, space of constant curvature

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1. Introduction

In 1950, A.V. Pogorelov proved the following rigidity result for closed convex surfaces in the Euclidean space E^3.

Theorem A ([1]). *Let F_1 and F_2 be a pair of closed convex surfaces in E^3 isometric with respect to their intrinsic metrics. Then there exists an isometry of the ambient Euclidean space E^3 that maps the surface F_1 onto the surface F_2.*

Notice that no regularity assumptions are required on the surfaces in the theorem above, only the convexity of surfaces is assumed. Under stronger regularity assumptions, Theorem A was previously proven by S. Cohn-Vossen in 1924 [2] and G. Herglotz in 1943 [3]. A.V. Pogorelov later extended Theorem A to general convex surfaces in the spherical space $\mathbb{S}^3$. Building on the results of A.V. Pogorelov, A.D. Alexandrov, and E.P. Sen'kin, A.D. Milka established analogous rigidity result in the hyperbolic (Lobachevsky) space $\mathbb{H}^3$. E.P. Sen'kin further generalized Pogorelov's theorem to Euclidean spaces of arbitrary dimension [5], but under additional assumptions on the regularity of hypersurfaces.

Theorem 1.1 ([5]). *Let F_1, F_2 be a pair of closed convex C^1-smooth hypersurfaces in the Euclidean space E^{n+1}. If F_1 and F_2 are isometric with respect to their intrinsic metrics, then there exists an isometry of the ambient space E^{n+1} that maps one hypersurface onto the other.*

In this paper, we prove Theorem 1.1 without any regularity assumption on hypersurfaces. More precisely, our goal is to establish the following result.

Theorem 1.1′. *Let F_1 and F_2 be a pair of closed convex hypersurfaces in the Euclidean space E^{n+1}, $n \geq 3$. If F_1 and F_2 are isometric with respect to their intrinsic metrics, then there exists a motion of E^{n+1} that maps F_1 onto F_2.*

The proof of this theorem proceeds through a sequence of steps based on the lemmas below.

We say that a hypersurface $F \subset E^{n+1}$ is *visible from a point* $Q \in E^{n+1} \setminus F$ if, for every point $P \in F$, the ray QP intersects F only at P. Furthermore, a point P is said to be *visible from the inside* if the ray QP forms an acute angle with the outer normal to the supporting hyperplane of F at P.

We also say that a pair of hypersurfaces is congruent if there exists a motion of E^{n+1} that maps one hypersurface to the other one.

Lemma 1.2 ([5]). *Let F_1 and F_2 be a pair of isometric convex hypersurfaces in E^{n+1}. Suppose that they are visible from points Q_1 and Q_2. Let L_1 and L_2 be the boundaries of F_1 and F_2 (if the hypersurfaces are closed, then instead of boundaries we use a pair of points $X_1 \in F_1$ and $X_2 \in F_2$ that correspond to each other under the isometry). Assume there exist hyperplanes P_1 through Q_1 and P_2 through Q_2 such that for each $i \in \{1, 2\}$ the hypersurface F_i lies entirely in one half-space determined by P_i. If the distances from the points Q_1 and Q_2 to the corresponding under the isometry points of the boundaries L_1 and L_2 are equal, then either the hypersurfaces F_1 and F_2 are congruent, or there exists a motion ϕ of E^{n+1} such that*

1. *$\phi(X_1) = X_2$ for some points $X_1 \in F_1$ and $X_2 \in F_2$ that correspond to each other under the isometry of the hypersurfaces; we keep the notation F_1 for $\phi(F_1)$;*
2. *there exits a point $Q \in E^{n+1}$ and neighborhoods U_i of X_i in F_i such that the neighborhoods are visible from Q from the inside;*
3. *for every corresponding under the isometry points $X \in U_1$ and $X \in U_2$, we have*
$$r_1(X) < r_2(X),$$
where r_i denotes the distance function from Q to the points of U_i.

For a general (not necessarily smooth) surface $F \subset E^3$, we say that F has *non-positive curvature* if for every point on F there exists a neighborhood in which one cannot cut out a cup.

Lemma 1.3 ([1, Ch. IV, §2, p. 213]). *Let F be a two-dimensional convex surface in E^3 given explicitly by*

$$z = z(x, y),$$

where x, y, z are orthogonal Cartesian coordinates in E^3. Denote by $\xi(x, y)$ the z-component of an infinitesimal bending field on F, and consider the surface Φ given explicitly by

$$z = \xi(x, y).$$

If F does not contain flat regions, then Φ has non-positive curvature everywhere. If F contains flat regions, then the curvature of Φ is non-positive everywhere except for these flat regions.

Let F be the hypersurface given by the position vector

$$R = \frac{1}{2}(r_1 + r_2), \tag{1.1}$$

where r_1 and r_2 are the position vectors of F_1 and F_2 in Lemma 1.2. By this lemma, for every X, the points $r_1(X) = P_1 \in F_1$ and $r_2(X) = P_2 \in F_2$ correspond to each other under the isometry between F_1 and F_2, and $r_1(X_0) = r_2(X_0) = P_0$ for some point P_0 that satisfies Lemma 1.2.

Under the additional assumption that the hypersurfaces F_1 and F_2 are C^1-smooth, it was proved by Sen'kin that the hypersurface F with the position vector $R = \frac{1}{2}(r_1 + r_2)$ is convex in some neighborhood of the point P_0. Furthermore, the following statement holds.

Lemma 1.4. *The vector field $\sigma := r_1 - r_2$ is an infinitesimal bending field on the hypersurface F. It is Lipshitz and satisfies the equation*

$$\langle dR, d\sigma\rangle = 0 \qquad \text{a.e. in the neighborhood of } P_0.$$

Lemma 1.4 generalizes Alexandrov's theorem for convex surfaces in E^3.

Let us define

$$E^3 := \operatorname{span}(e_1, e_2, n),$$

where e_1, e_2 are tangent vectors and n is the normal vector to F at P_0. The intersection $F \cap E^3 =: F^2$ is a closed convex surface in E^3. We will work now in the subspace E^3. In the neighborhood of P_0 the surface F^2 is given explicitly by $z = z(x, y)$. Let $z = \xi(x, y)$ be the z-component of an infinitesimal bending field along F^2. The function $z = \xi(x, y)$ assumes its minimum at P_0. For sufficiently small $\varepsilon > 0$, the plane $z = \varepsilon$ cuts out a cap from the surface $z = \xi(x, y)$. This contradicts Pogorelov's Lemma 1.3. Therefore, $r_1 = r_2$, and the hypersurfaces F_1 and F_2 coincide. This completes the proof of Sen'kin's Theorem 1.1.

Now we will show that the hypersurface F is convex. After that we will prove Theorem 1.1′ in the same way as Theorem 1.1.

2. Convex combination of isometric hypersurfaces

In this section, we discuss some facts about convex combinations of convex hypersurfaces in E^4.

At every point of a convex hypersurface in E^4 there exists a well-defined tangent cone. This cone is a convex hypersurface as well. Let V^n be a strongly convex cone in the Euclidean space E^{n+1}; a convex cone is called *strongly convex* if there exists a supporting hyperplane of the cone through its vertex O that intersects the cone only at O.

It is well known that a tangent cone V^3 of a convex hypersurface $F^3 \subset E^4$ has one of the following forms:

1. V^3 is a strongly convex cone in E^4;
2. $V^3 = V^2 \times E^1$ is the metric product of a strongly convex cone V^2 in E^3 and a Euclidean line E^1;
3. $V^3 = V^1 \times E^2$ is the metric product of a strongly convex cone V^1 in E^2 and a Euclidean plane E^2;
4. $V^3 = E^3$ is a Euclidean space E^3.

If points $P_1 \in F_1$ and $P_2 \in F_2$ correspond to each other under the isometry of isometric convex hypersurfaces F_1 and F_2, then the tangent cones of the hypersurfaces at these points are isometric too.

Lemma 2.1. *Let F_1 and F_2 be a pair of isometric convex hypersurfaces in E^4.*

I. *Suppose that the tangent cone $K(P_1)$ at a point $P_1 \in F_1$ has the form (1). Then for the corresponding under the isometry point $P_2 \in F_2$ the tangent cone $K(P_2)$ has the same form (1); and furthermore, the cones $K(P_1)$ and $K(P_2)$ are congruent.*

II. *If the cone $K(P_1)$ has the form (2), i.e., $K(P_1) = V_1^2 \times E_1^1$, then $K(P_2)$ has the same form $K(P_2) = V_2^2 \times E_2^1$; and furthermore, the cones V_1^2 and V_2^2 are isometric. The edges E_1^1, E_2^1 correspond to each other under the isometry of $K(P_1)$ and $K(P_2)$.*

Proof. I. Suppose $K(P_2)$ has one of the forms (2), (3), (4). In each case, we can choose a straight segment $\gamma_2 \subset K(P_2)$ such that P_2 lies in the interior of γ_2. Since $K(P_1)$ and $K(P_2)$ are isometric, for $K(P_1)$ there exists a corresponding shortest line $\gamma_1 \subset K(P_1)$ through P_1. The curve γ_1 is isometric to γ_2. The point P_1 splits γ_1 into two straight segments γ_1^+ and γ_1^- with P_1 being their common boundary point.

Let $E^3 = \operatorname{span}(\gamma_1^+, \gamma_1^-, \ell)$, where ℓ is a ray inside the cone $K(P_1)$ which does not belong to the plane $\operatorname{span}(\gamma_1^+, \gamma_1^-)$. The intersection $K(P_1) \cap E^3$ is a strongly convex cone in E^3. For this cone, γ_1 is the shortest line in the cone, it passes through P_1, and this point lies in the interior of γ_1. This contradicts to the fact that on a strongly convex cone in E^3 a shortest line cannot go through the vertex of the cone.

Let us show that $K(P_1)$ and $K(P_2)$ are congruent, i.e., there exists a motion of the Euclidean space E^4 that maps one cone onto the other. Let S_1^3 and S_2^3 be the unit spheres with the centers at the points P_1 and P_2, respectively. Then $\tilde{F}_i^2 = K(P_i) \cap S_i^3$, $i \in \{1, 2\}$, are isometric closed convex surfaces in open hemispheres of S_1^3, S_2^3. By moving the spheres, if necessary, we can assume that $\tilde{F}_1^2$ and $\tilde{F}_2$ belong to the same spherical space, and hence we can apply to them the following theorem due to A.V. Pogorelov.

Theorem B ([1])**.** *Closed isometric convex surfaces in the spherical space S^3 are congruent.*

This completes the proof of Part I of Lemma 2.1.

II. The proof of Part II is similar to that of Part I. □

Lemma 2.2. *Let F_1 and F_2 be a pair of isometric convex hypersurfaces in E^4. Suppose the tangent cone at a point $P_1 \in F_1$ has the form (3) or (4). Then the tangent cone $K(P_2)$ at the corresponding under the isometry point $P_2 \in F_2$ has the same form (3) or (4) as well. Furthermore, the following three possibilities can occur:*

a) *both tangent cones are dihedral angles, $K(P_1) = V_1^1 \times E_1^1$ and $K(P_2) = V_2^1 \times E_2^1$;*

b) *one tangent cone is a hyperplane, whereas the other one is a dihedral angle;*

c) *both tangent cones are hyperplanes.*

Let G_1, G_2 be small neighborhoods of the points $P_1 \in F_1$ and $P_2 \in F_2$, $P_1 = P_2 = P_0$, which satisfy the assumptions of Lemma 1.2. Consider the tangent cones $K(P_1)$ and $K(P_2)$. The following cases can occur:

I. $K(P_1) = V^3$. Then the cones $K(P_1)$ and $K(P_2)$ coincide.

II. $K(P_1) = V_1^2 \times E_1^1$. Then the cones $K(P_1)$ and $K(P_2)$ coincide too. By Lemma 1.2, we get $V_1^2 \subseteq V_2^2$. By the isometry of V_1^2 and V_2^2, $V_1^2 = V_1^2$ and the lines E_1^1 and E_2^1 coincide.

III. a) If both tangent cones are dihedral angles, then it follows from Lemma 1.2 that the edges E_1^2, E_2^2 correspond to each other under the isometry and coincide, and one dihedral angle lies inside the other one.

b) If both tangent cones are hyperplanes, then they coincide.

c) If one cone is a hyperplane and the other cone is a dihedral angle, then the argument is similar to case a).

In every case, the linear combination of the cones at the point P_0 is a convex dihedral angle.

Let us treat the cases separately.

I. $K(P_1) = K(P_2) = V^3$.

I.1. Let $(X_1^n) \subset F_1$ and $(X_2^n) \subset F_2$ be sequences of the corresponding under the isometry cone points such that $X_1^n \to P_0$ and $X_2^n \to P_0$ as $n \to \infty$, and $K(X_1^n) = V_n^3$. Denote by K_1^0 and K_2^0 the limit cones for the sequences $K_1(X_1^n)$ and $K_2(X_2^n)$. By the construction, K_1^0 and K_2^0 are isometric supporting cones to F_1 and F_2 at P_0.

By Lemma 2.1, for each n, we have

$$K_1(X_1^n) = A_n K_2(X_2^n) + a_n,$$

where a_n is a vector and A_n is an orthogonal matrix. Then $a_n \to 0$ and $A_n \to A_0$ as $n \to \infty$, where A_0 is an orthogonal matrix. Since $K_1^0 = K_2^0$, we obtain $K_1^0 = A_0 K_2^0$, and thus $A_0 = I$ is the identity matrix. For large n, the matrix $I + A_n$ is non-degenerate, and the convex combination of the cones $K_1(X_1^n)$ and $K_2(X_2^n)$ is the cone $K(X^n) = (I + A_n) \cdot K_2(X_2^n) + a_n$. Thus, we obtain that $K(X^n)$ is a non-degenerate affine image of $K(X_2^n)$, and hence it is convex.

I.2. Let $K_1(X_1^n) = V_1^2(n) + E_1^1(n)$, $K_2(X_2^n) = V_2^2(n) + E_2^1(n)$. If $K_1^0 = V_1^2 \times E_1^1$ and $K_2^0 = V_2^2 \times E_2^1$, then the isometric directions $\ell_1^0 \in V_1^2$ and $\ell_2^0 \in V_2^2$ belong to the tangent cones $K(P_1) = K(P_2)$. Therefore we have $K_1^0 = K_2^0$, $V_1^2 = V_2^2$, $E_1^1 = E_2^1$. The curvature of V_1^2 is greater than some $\alpha_0 > 0$. The angle between any pair of isometric directions in the cones $V_1^2(n)$ and $V_2^2(n)$ is less than $\epsilon(n)$, where $\epsilon(n) \to 0$ as $n \to \infty$. The curvature at the vertices is at least $\theta_0 > 0$, and a ball ω belongs to both cones. We will show now that for sufficiently large n the convex combination of cones $K_1(X_1^n)$ and $K_2(X_2^n)$ is again a convex cone. To this end, it suffices to show that the cone

$$K(X^n) = K_1(X_1^n) + K_2(X_2^n)$$

is locally convex. Equivalently, we need to demonstrate that through every two-dimensional generator t_0 of $K(X^n)$ it is possible to draw a hyperplane such that all generators close to t_0 lie in the half-space that contains the ball ω. Assume the contrary, i.e., for each n, there exists a generator t_0^n that does not satisfy the locally convex condition. Let $t_1^n \in K_1(X_1^n)$ be the corresponding generator of $K_1(X_1^n)$. The sequence of generators t_1^n converges to the generator t_1^0 of the convex cone K_1^0. The generators t_1^n are the metric products of generators $\ell_1^n \in V_1^2(n)$ with $E_1^1(n)$. For each n, let A_1^n be the point on ℓ_1^n at distance 1 from the vertex of the cone $V_1^2(n)$, and let $\mathcal{D}_1^n$ be the tangent dihedral angle at the point A_1^n for the cone $K_1(X_1^n)$. Introduce the same elements t_2^n, ℓ_2^n, A_2^n, $\mathcal{D}_2^n$ for the cone $K_2(X_2^n)$. The two-dimensional edges of $\mathcal{D}_1^n$ and $\mathcal{D}_2^n$ are corresponding to each other under the isometry. For sufficiently large n, the convex combination of $\mathcal{D}_1^n$ and $\mathcal{D}_2^n$ is a dihedral angle $\mathcal{D}^n$. By the construction, the ball ω lies inside $\mathcal{D}^n$. There exists a supporting hyperplane Π to $\mathcal{D}^n$ that passes through the edge of $\mathcal{D}^n$. Now, we follow Pogorelov's proof of [1, Lemma 1, pp. 137–136].

Let $\bar{n}$ be the normal to Π. When translated to a point A_i^n, the vector $\bar{n}$ is directed inside the cone $K_i(X_i^n)$ for every $i \in \{1, 2\}$. Connect the point A_1^n by the shortest line γ_1^n to a point $B_1^n \in V_1^2(n)$ near A_1^n. Let $r_1(s)$ be the position vector of γ_1^n, where s is the arc-length parameter on γ_1^n chosen such that $s = 0$ at the point A_1^n. Similarly, let $r_2(s)$ be the position vector of the corresponding under the isometry shortest line $\gamma_2^n \subset V_2^2(n)$. At $s = 0$, we get the following:

$$\frac{d}{ds} \langle r_1 + r_2, \bar{n} \rangle \geq 0. \tag{2.1}$$

By Liberman's theorem [1, p. 58], inequality (2.1) holds true for all s along the shortest line γ_1^n. Furthermore, by integrating this inequality, we obtain that all points of the cone $K(X^n)$ close to the image of the point A_1^n lie on one side of the supporting hyperplane with the inner normal $\bar{n}$. This implies that the cone $K(X^n)$ is locally convex.

Take small neighborhoods of the point $P_0 = P_1 = P_2$ in the hypersurfaces F_1 and F_2. Let F be the convex combination of F_1 and F_2. The position vector of F is $r = (r_1 + r_2)/2$, where r_i is the position vector of F_i. It follows from the discussion above that there exists a neighborhood of $P_0 \in F$ such that F is a convex hypersurface. The vector field $\sigma = r_1 - r_2$ is an infinitesimal bending vector field of F, i.e., one has $\langle dr, d\sigma\rangle = 0$. We then proceed as in Sen'kin's original proof, and this completes the proof of the desired uniqueness theorem for closed convex hypersurfaces in the Euclidean space E^4.

I.3. Let

a) $K_1(X_1^n) = V_1^1(n) \times E_1^2(n)$, $K_2(X_2^n) = V_2^1(n) \times E_1^2(n)$.
b) $K_1(X_1^n) = V_1^1(n) \times E_1^2(n)$, $K_2(X_2^n) = E^3(n)$.
c) $K_1(X_1^n) = E_1^3(n)$, $K_2(X_2^n) = E_2^3(n)$.

In these cases, the dihedral angles or supporting hyperplanes are equal. The convex combination of the cones $K_1(X_1^n)$ and $K_2(X_2^n)$ has the following form, respectively:

a) a hyperplane,
b) a dihedral angle,
c) a convex cone.

In the same way as proved above, there exist neighborhoods of the point P_0 in F_1 and F_2 such that the convex combination of F_1 and F_2 within these neighborhoods is a convex hypersurface F.

II. $K_1(P_1) = K_2(P_2) = V_2 \times E^1$.

III. a) If $K_1(P_1) = E^3$ and $K_2(P_2) = E^3$, then K_1^0 and K_2^0 coincide with the tangent hyperplane.

b) If $K_1(P_1) = V_1^1 \times E_1^2$ and $K_2(P_2) = V_2^1 \times E_2^2$, then the edges E_1^2 and E_2^2 coincide and correspond to each other under the isometry of the cones. Furthermore, one dihedral angle lies inside the other one. In this case, the cones K_1^0 and K_2^0 are contained inside the cones $K_1(P_1)$ and $K_2(P_2)$. Similarly to the case I, we can prove that there exist neighborhoods of the points $P_1 \in F_1$ and $P_2 \in F_2$ such that the convex combination of F_1 and F_2 is a convex surface F.

3. Proof of Theorem 1.1′

In this section, we will prove the uniqueness theorem for isometric closed convex hypersurfaces in E^n without any regularity assumption (Theorem 1.1′).

We need a concept of the *Pogorelov map* [1]. Let F_1 and F_2 be isometric closed convex hypersurfaces in the open hemisphere of the spherical space $S^n \subset E^{n+1}$. Let $x^0, x^1, \dots, x^n$ be the Cartesian orthogonal coordinates in E^{n+1}, and let S^n be a sphere centered at the origin. We assume that F_1 and F_2 have the same orientation and belong to the same hemisphere $x^0 > 0$. Let r_1 and r_2 be the position vectors of F_1 and F_2 parameterized such that points corresponding under the isometry have the same coordinates. Finally, let Φ_1 and Φ_2 be the hypersurfaces in E^n defined by the position vectors

$$R_1 := \frac{r_1 - e_0\,\langle r_1, e_0\rangle}{\langle e_0, r_1 + r_2\rangle}, \quad R_2 := \frac{r_2 - e_0\,\langle r_2, e_0\rangle}{\langle e_0, r_1 + r_2\rangle},$$

where e_0 is the unit coordinate vector corresponding to x^0. For $n = 4$, A.V. Pogorelov proved that Φ_1 and Φ_2 are isometric closed convex hypersurfaces in E^4. Notice that this result is true for any n and the proof is similar to that of Pogorelov for $n = 4$. Thus, the uniqueness theorem in S^4 follows from the uniqueness theorem in E^4.

Now we prove Theorem 1.1′ in the hyperbolic space $\mathbb{H}^n$, $n = 4$. In 1980, A.D. Milka proved Theorem 1.1′ in the case of $\mathbb{H}^3$ [4]. He used E.P. Sen'kin's idea of the proof of Theorem 1.1. Specifically, it is possible to move the surfaces F_1 and F_2 in such a way that they satisfy Lemma 1.2. For hyperbolic spaces, this means that for some point O the surfaces F_1 and F_2 are visible from the different sides. Then their images under Pogorelov's map in Euclidean space satisfy Lemma 1.2. Do Carmo and Warner proved the uniqueness of closed regular convex hypersurfaces in S^n [6]. Gorsij generalized this theorem to isometric closed convex C^1-smooth hypersurfaces in S^n [7]. It was also proved that the images of isometric convex hypersurfaces in S^n under the Pogorelov map are convex hypersurfaces in the Euclidean space E^n if the hypersurfaces in the sphere can be seen from the convexity side [1]. Milka proved a similar result for isometric closed convex hypersurfaces in the hyperbolic space.

Proof of Theorem 1.1′. The proof proceeds by induction on the dimension n. Suppose the statement holds true for E^n, S^n, $\mathbb{H}^n$. Let us show how to prove it for E^{n+1}, S^{n+1}, $\mathbb{H}^{n+1}$.

For convex hypersurfaces $F_s^n \subset E^{n+1}$, $s \in \{1, 2\}$, their tangent convex cones have the form

$$K = V^{n-i} \times E^i, \quad i \in \{0, \dots, n\},$$

where V^{n-i} is a strongly convex cone in E^{n-i+1}.

1) Let $K_1 = V_1^{n-i} \times E_1^i$ $(i \leq n-3)$ be a convex cone in E^{n+1}, and let K_2 be an isometric convex cone in E^{n+1}. Then $K_2 = V_2^{n-i} \times E_2^i$ is congruent to K_1. In the proof we follow the same steps as those of Lemma 2.1, and we use the uniqueness of isometric closed convex hypersurfaces in E^{n-1}, S^{n-1}.

2) Let $K_1 = V_1^2 \times E_1^{n-2}$, and let K_2 be an isometric convex cone in E^{n+1}. Then $K_2 = V_2^2 \times E_2^{n-2}$. The cones $K_1, K_2 \subset E^3$ are isometric convex cones.

3) Let $K_1 = V_1^1 \times E_1^{n-1}$. Then $K_2 = V_2^1 \times E_2^{n-1}$ or $K_2 = E^n$.

We prove Theorem 1.1′ similarly to the case of $E^4, S^4, \mathbb{H}^4$ by induction. □

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Однозначна визначеність замкнутих опуклих гіперповерхонь в багатомірних просторах сталої кривини

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У 1972 році Є.П. Сенькін узагальнив знамениту теорему О.В. Погорєлова про однозначну визначеність замкнутих опуклих поверхонь їх внутрішньою метрикою в тримірному евклідовому просторі E^3 на випадок багатомірного евклідового простору E^{n+1} за деяких додаткових умов щодо гладкості гіперповерхонь. У цій статті ми позбавляємось згаданих умов і встановлюємо теорему про однозначну визначеність для довільних замкнутих опуклих гіперповерхонь в E^{n+1}, $n \geq 3$. Аналогічні результати також одержані і в інших модельних просторах сталої кривини.

Ключові слова: однозначна визначеність, опукла гіперповерхня, простір сталої кривини