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Optimal Control Problems for Evolutionary Variational Inequalities with Volterra-Type Operators

Mykola Bokalo and Olha Sus

In this paper, we consider an optimal control problem for a class of evolution inclusions with Volterra-type operators. The existence and uniqueness results for the initial value problem for such inclusions were obtained in our previous work. Here we establish the existence of a solution to the stated optimal control problem under some hypothesis on data-in. The motivation for this work comes from the optimal control problems for variational inequalities arising in the study of injection molding processes, contact mechanics, processes of electro-wetting on dielectrics, and others.

Key words: parabolic variational inequality, evolutionary variational inequality, Volterra-type operator, optimal control

Mathematical Subject Classification 2020: 34H05, 35K86, 49J20, 49J21, 49J27, 49J40

1. Introduction

In this paper, we consider optimal control problems for evolutionary (particularly, parabolic) variational inequalities (subdifferential inclusions) with history-dependent operators, the so-called Volterra-type operators.

Optimal control problems for variational inequalities have been a subject of interest in the optimal control community starting from the 1980s. The motivation for this study comes from broad interesting applications. For instance, in work [11], one can find its application in the problem related to the principles of electrowetting on dielectrics. Also, such problems appear at the simulation of various problems related to injection molding processes, contact mechanics, etc. (see, e.g., [19]). Furthermore, it has its application in problems in economics, finances, optimization theory and many others (see, e.g., [8, 15, 22] and reference therein).

The optimal control of evolutionary problems has been extensively studied in the literature. In the book [17], the optimal control of systems governed by partial differential equations was considered. The existence and approximation of optimal solutions as well as the necessary optimality conditions for parabolic control problems were studied, for instance, in [1, 9] and others. In [5], the optimal control problems for systems governed by parabolic equations without initial conditions with controls in the coefficients were considered. The authors proved the existence of solutions to an optimal control Fourier problem (i.e., problem without initial conditions) for parabolic equations, where the control functions occur in the coefficients of the state equation. They discussed the well-posedness of the problem and gave a necessary optimality condition in the form of a generalized principle of Lagrange multipliers. Also, in [7], the authors studied optimal control problems governed by a class of semiliniear evolution equations including the so-called equations with memory.

In the beginning of 1980s, the first papers on optimal control for variational inequalities appeared. In particular, such problems were intensively studied in [3,25]. Also, optimal control problems for some variational and hemivariational inequalities were considered in [13,18]. Optimal control problems for the subdifferential evolution inclusions were examined in many works, see, e.g., [14,20,21] and references therein. More precisely, in [20], N. Papageorgiou studied the Volterra integro-differential evolution inclusions of nonconvolution type with time dependent unbounded operators and with both convex and nonconvex multivalued perturbations. In [16], the optimal control of parabolic variational inequalities was studied for the case where spatial domain is not necessarily bounded. In [6], the existence, uniqueness and convergence of optimal control problems associated with parabolic variational inequalities of the second kind were studied. In [26], the variational stability of optimal control problems involving subdifferential operators was studied. The evolutionary variational-hemivariational inequalities with applications to dynamic viscoelastic contact mechanics were considered in [12]. There was studied a complicated variational-hemivariational inequality of parabolic type with history-dependent operators.

The most recent paper, known to us, is [19], where S. Migórski investigated the optimal control of history-dependent evolution inclusions. In this paper, a class of subdifferential evolutionary inclusions involving history-dependent operators was studied. The main idea of this paper is that the existence and uniqueness results for such problems were proved by removing the smallness condition. Also, the continuous dependence of the solution to these inclusions on the second member and the initial condition was examined and the Bolza-type optimal control problem was studied. More precisely, for given $A : (0,T) \times V \to V^*$, $\psi : (0,T) \times V \to \mathbb{R}$, $f : (0,T) \to V^*$ and $w_0 \in V$, where (V, H, V^*) forms an evolution triple of spaces, the author at first considered the following Cauchy problem.

Problem. Find $w \in W$ such that

$$w'(t) + A(t, w(t)) + \partial\psi(t, w(t)) \ni f(t) \quad \text{for a.e.} \quad t \in (0,T),$$
$$w(0) = w_0.$$

Here, $\partial\psi$ denotes the Clarke generalized gradient of locally Lipschitz function $\psi(t, \cdot)$ and $W = \{w \in L^2(0,T;V) \mid w' \in L^2(0,T;V^*)\}$.

It was shown that under some hypothesis, the stated problem has a unique solution. The author used this result to examine the evolution inclusion of subdifferential type with history-dependent operators and the optimal control problem associated with this evolution inclusion.

In the present paper, we turn our attention to the optimal control problem for a class of evolution inclusions with Volterra-type operators, which are history-dependent. The stated problem is considered in the framework of an evolutionary triple of spaces. The existence and uniqueness results for the initial value problem of such inclusions have been proved in [4]. As our main result, we establish the existence of a solution to the stated optimal control problem under some hypothesis on data-in. Based on the preceding papers mentioned above, we can conclude that optimal control problems for subdifferential inclusions with Volterra-type operators have not been investigated yet. It motivates us to study this kind of problems here.

The outline of the paper is as follows. In Section 2, we formulate main notations and auxiliary facts. The statement of the initial value problem for evolutionary subdifferential inclusions and the formulation of the results on the existence and uniqueness of its solutions is given in Section 3. The main results of this paper are stated in Section 4. In Section 5, we prove the main result. Comments on the main result are given in Section 6.

2. Preliminaries

Let $T > 0$, $p \geq 2$ be arbitrary constants and their values be fixed throughout the paper. By p', we denote conjugated to p, i.e., $1/p + 1/p' = 1$.

Let V be a separable reflexive Banach space with the norm $\|\cdot\|$, H be a Hilbert space with the scalar product $(\cdot,\cdot)$ and the norm $|\cdot|$. Assume that embedding $V \subset H$ is dense, continuous and compact.

By V' and H', we denote the dual spaces to V and H. We assume, after an appropriate identification of functionals, that the space H' is a subspace of V'. Identifying H' with H by the Riesz–Fréchet representation theorem, one usually writes

$$V \subset H \subset V', \tag{2.1}$$

where all embeddings are dense, continuous and compact. Notice that in this case, $\langle g, v\rangle_V = (g, v)$ for every $v \in V$, $g \in H$, where $\langle \cdot,\cdot\rangle_V$ is the scalar product on the dual pair $\big[V', V\big]$. Therefore, we will use the notation $(\cdot,\cdot)$ instead of $\langle\cdot,\cdot\rangle_V$.

Now, we introduce some functional spaces and spaces of distributions. Let X be an arbitrary Banach space with the norm $\|\cdot\|_X$. By $C([0,T];X)$, we mean the Banach space of continuous functions $w : [0,T] \to X$ with the norm $\|w\|_{C(0,T;X)} := \max\limits_{t\in[0,T]} \|w(t)\|_X$. By $L^q(0,T;X)$, where $q \in [1,\infty)$, we denote the Banach space of (classes) measurable functions $w : (0,T) \to X$ such that $\|w(\cdot)\|_X \in L^q(0,T)$ with the norm $\|w\|_{L^q(0,T;X)} := \Big(\int_0^T \|w(t)\|^q\,dt\Big)^{1/q}$. By $L^\infty(0,T;X)$, we denote the Banach space of (classes) measurable functions $w : (0,T) \to X$ such that $\|w(\cdot)\|_X \in L^\infty(0,T)$ with the norm $\|w\|_{L^\infty(0,T;X)} := \operatorname*{ess\,sup}\limits_{t\in(0,T)} \|w(t)\|_X$. By $D'(0,T;X)$, we mean the space of distributions on $D(0,T)$

with values on X, i.e., the space of linear continuous (in weak topology on X) functionals on $D(0,T)$ with values on X. Hereafter $D(0,T)$ is the space of test functions, that is, the space of infinitely differentiable on $(0,T)$ functions with compact supports, equipped with the corresponding weak topology.

By (2.1), it is easy to see that the spaces $L^q(0,T;V)$, $L^2(0,T;H)$, $L^{q'}(0,T;V')$, where $q\in[1,\infty]$, $1/q+1/q'=1$, can be identified with the corresponding subspaces of $D'(0,T;V')$. In particular, this allows us to talk about derivatives w' of functions w from $L^q(0,T;V)$ and $L^2(0,T;H)$ in the sense of distributions $D'(0,T;V')$ and belonging of these derivatives to $L^{q'}(0,T;V')$ or $L^2(0,T;H)$.

Let us define the spaces

$$H^1(0,T;H) := \{w\in L^2(0,T;H)\mid w'\in L^2(0,T;H)\},$$
$$W^1_p(0,T;V) := \{w\in L^p(0,T;V)\mid w'\in L^{p'}(0,T;V')\}.$$

From the known results (see [10, pp. 177–179]), it follows that $H^1(0,T;H)\subset C([0,T];H)$ and $W^1_p(0,T;V)\subset C([0,T];H)$. Moreover, for every w in $H^1(0,T;H)$ or $W^1_p(0,T;V)$, the function $t\mapsto |w(t)|^2$ is absolutely continuous on $[0,T]$ and the following equality holds:

$$\frac{d}{dt}|w(t)|^2 = 2(w'(t),w(t)) \quad \text{for a.e.} \quad t\in[0,T]. \tag{2.2}$$

In this paper, below we use the well-known facts.

Proposition 2.1 ([27, p. 173])**.** *Let Y be a Banach space with the norm $\|\cdot\|_Y$, and $\{v_k\}_{k=1}^\infty$ be the sequence of elements of Y which is weakly or $*$-weakly convergent to v in Y. Then $\varliminf\limits_{k\to\infty}\|v_k\|_Y\ge\|v\|_Y$.*

Proposition 2.2 ([2], Aubin Theorem)**.** *Suppose that q,r are some numbers from interval $(1,\infty)$, and B_0,B_1,B_2 are Banach spaces such that $B_0\overset{c}{\subset}B_1\circlearrowleft B_2$ (here $\overset{c}{\subset}$ means compact embedding and $\circlearrowleft$ means continuous embedding). Then*

$$\{w\in L^q(0,T;B_0)\mid w'\in L^r(0,T;B_2)\}\overset{c}{\subset}\big(L^q(0,T;B_1)\cap C([0,T];B_2)\big).$$

We understand this embedding as follows: if the sequence $\{w_m\}_{m\in\mathbb{N}}$ is bounded in $L^q(0,T;B_0)$ and the sequence $\{w'_m\}_{m\in\mathbb{N}}$ is bounded in $L^r(0,T;B_2)$, then there exists a function $w\in L^q(0,T;B_1)\cap C([0,T];B_2)$ and a subsequence w_{m_j} of the sequence $\{w_m\}$ such that $w_{m_j}\underset{j\to\infty}{\longrightarrow} w$ in $C([0,T];B_2)$ and strongly in $L^q(0,T;B_1)$.

3. Initial value problem for evolutionary subdifferential inclusions

Let $\Phi: V\to(-\infty,+\infty]$ be a proper functional, i.e., $\mathrm{dom}(\Phi):=\{v\in V\mid \Phi(v)<+\infty\}\ne\varnothing$, which satisfies the following conditions:

($\mathcal{A}_1$) the functional Φ is *convex*,
i.e., $\Phi\big(\alpha v+(1-\alpha)w\big)\le\alpha\Phi(v)+(1-\alpha)\Phi(w)$, $v,w\in V$, $\alpha\in[0,1]$;

($\mathcal{A}_2$) the functional Φ is *lower semi-continuous*,
i.e., $v_k \underset{k\to\infty}{\longrightarrow} v$ in $V \implies \varliminf\limits_{k\to\infty} \Phi(v_k) \geq \Phi(v)$.

Let us recall (see, for example, [24]) that the *subdifferential* of functional Φ is a map $\partial\Phi : V \to 2^{V'}$, defined as follows:

$$\partial\Phi(v) := \{v^* \in V' \mid \forall\, w \in V \quad \Phi(w) \geq \Phi(v) + (v^*, w - v)\}, \quad v \in V,$$

and the *domain* of the subdifferential $\partial\Phi$ is the set $D(\partial\Phi) := \{v \in V \mid \partial\Phi(v) \neq \varnothing\}$. We identify the subdifferential $\partial\Phi$ with its graph assuming that $[v, v^*] \in \partial\Phi$ if and only if $v^* \in \partial\Phi(v)$, i.e., $\partial\Phi = \{[v, v^*] \mid v \in D(\partial\Phi),\ v^* \in \partial\Phi(v))\}$. R. Rockafellar (see [23, Theorem A]) proves that the subdifferential $\partial\Phi$ is a *maximal monotone operator*, that is,

$$(v_1^* - v_2^*, v_1 - v_2) \geq 0, \quad [v_1, v_1^*],\ [v_2, v_2^*] \in \partial\Phi, \tag{3.1}$$

and for every element $[v_1, v_1^*] \in V \times V'$, we have the implication

$$\big(\forall\, [v_2, v_2^*] \in \partial\Phi \quad (v_1^* - v_2^*, v_1 - v_2) \geq 0\big) \ \Rightarrow\ [v_1, v_1^*] \in \partial\Phi.$$

Additionally, assume that the following conditions hold:

($\mathcal{A}_3$) there exists a constant $K > 0$ such that

$$\Phi(v) \geq K\|v\|^p, \quad v \in \mathrm{dom}(\Phi),$$

moreover, $\Phi(0) = 0$;

($\mathcal{B}$) $B : C([0,T]; H) \to L^\infty(0,T; H)$ is an operator such that, for a.e. $t \in (0,T)$ and for any $w_1, w_2 \in C([0,T]; H)$, the following inequality holds:

$$|B(w_1)(t) - B(w_2)(t)| \leq L \int_0^t |w_1(s) - w_2(s)|\, ds,$$

where $L \geq 0$ is a constant; moreover, $B(0) = 0$.

The operator B is called a *Volterra-type operator*.

Remark 3.1. Condition ($\mathcal{A}_3$) implies that $\Phi(v) \geq \Phi(0) + (0, v - 0)$, $v \in V$, hence $[0,0] \in \partial\Phi$.

Remark 3.2. It is easy to verify that condition ($\mathcal{B}$) implies

$$|B(w)(t)| \leq L \int_0^t |w(s)|\, ds \tag{3.2}$$

for a.e. $t \in (0,T)$ and for any $w \in C([0,T]; H)$.

Let us consider the *evolutionary subdifferential inclusion*

$$y'(t) + \partial\Phi\big(y(t)\big) + B(y)(t) \ni f(t), \quad t \in (0,T), \tag{3.3}$$

where $f : (0,T) \to V'$ is a given measurable function, $y : [0,T] \to V$ is an unknown function.

Let conditions ($\mathcal{A}_1$)–($\mathcal{A}_3$), ($\mathcal{B}$) hold, and $f \in L^{p'}(0,T; V')$.

Definition 3.3. The function y is a solution of variational inequality (3.3) if it satisfies the following conditions:

1) $y \in W_p^1(0,T;V)$ (then $y \in C([0,T];H)$);

2) $y(t) \in D(\partial\Phi)$ for a.e. $t \in (0,T)$;

3) there exists a function $g \in L^{p'}(0,T;V')$ such that, for a.e. $t \in (0,T)$, we have $g(t) \in \partial\Phi\big(y(t)\big)$, and

$$y'(t) + g(t) + B(y)(t) = f(t) \quad in \quad V'. \tag{3.4}$$

By $\mathbf{P}(\Phi, B, f, y_0)$, we denote the *problem* of finding a solution y for variational inequality (3.3) that satisfies the condition

$$y(0) = y_0, \tag{3.5}$$

where $y_0 \in H$ is given.

Remark 3.4. The problem $\mathbf{P}(\Phi, B, f, y_0)$ can be replaced by the following one. Let K be a convex closed set in V, $A : V \to V'$ be a monotone bounded and semi-continuous operator such that $(A(v), v) \geq \widetilde{K}\|v\|^p$ for all $v \in V$, where $\widetilde{K} = \mathrm{const} > 0$, and $f \in L^{p'}(0,T;V')$, $y_0 \in H$. The task is to find a function $y \in W_p^1(0,T;V)$ such that $y(0) = y_0$ and, for a.e. $t \in (0,T)$, we have $y(t) \in K$ and for all $v \in K$,

$$(y'(t) + A(y(t)) + B(y)(t), v - y(t)) \geq (f(t), v - y(t)).$$

This is called the evolutionary variational inequality. For more information, see Section 6.

In [4], we obtained the following results.

Theorem 3.5 ([4, Theorem 1]). *Let conditions $(\mathcal{A}_1)$–$(\mathcal{A}_3)$, $(\mathcal{B})$ hold, and $f \in L^{p'}(0,T;V')$, $y_0 \in H$. Then the problem $\boldsymbol{P}(\Phi, B, f, y_0)$ has no more than one solution.*

Theorem 3.6 ([4, Theorem 2]). *Let the following conditions hold: $(\mathcal{A}_1)$–$(\mathcal{A}_3)$, $(\mathcal{B})$, and*

$(\mathcal{F})$ $\quad f \in L^2(0,T;H), \quad y_0 \in \mathrm{dom}(\Phi)$.

Then the problem $\boldsymbol{P}(\Phi, B, f, y_0)$ has a unique solution, it belongs to $L^\infty(0,T;V) \cap H^1(0,T;H)$ and satisfies the following estimate:

$$\max_{t\in[0,T]} |y(t)|^2 + \operatorname*{ess\,sup}_{t\in[0,T]} \|y(t)\|^p + \int_0^T |y'(t)|^2\,dt \leq C_1 \left(|y_0|^2 + \Phi(y_0) + \int_0^T |f(t)|^2\,dt \right), \tag{3.6}$$

where C_1 is a positive constant, which depends only on K, L and T.

4. Statement of the optimal control problem and the main result

Let H^* be a Hilbert space with the scalar product $(\cdot,\cdot)_{H^*}$ and the corresponding norm $\|\cdot\|_{H^*} := \sqrt{(\cdot,\cdot)_{H^*}}$. We consider the space of controls

$$U := \left\{ u \in L^2_{\mathrm{loc}}(0,T;H^*) \;\middle|\; \int_0^T \omega(t)\|u(t)\|^2_{H^*} dt < \infty \right\},$$

where $\omega \in C(0,T)$, and $\omega(t) > 0$ for all $t \in (0,T)$. It is a Hilbert space with the scalar product and the norm

$$(u_1,u_2)_U := \int_0^T \omega(t)(u_1(t),u_2(t))_{H^*}\,dt, \quad \|u\|_U := \left(\int_0^T \omega(t)\|u(t)\|^2_{H^*}\,dt\right)^{1/2}$$

for all $u_1, u_2, u \in U$.

Let $U_\partial \subset U$ be a convex closed set, which is called the *set of admissible controls*.

Further, we will assume that the following conditions hold: $(\mathcal{A}_1)$–$(\mathcal{A}_3)$, $(\mathcal{B})$, $(\mathcal{F})$, and

$(\mathcal{M})$ $\quad M : U \to L^2(0,T;H)$ is a linear continuous operator.

For a given control $u \in U_\partial$, the state $y(t)$, $t \in [0,T]$, (which can also be denoted by $y(u)$ or $y(\cdot;u)$) of the control evolutionary system is described by the solution of the evolutionary variational inequality

$$y'(t) + \partial\Phi\big(y(t)\big) + B(y)(t) \ni f(t) + Mu(t), \quad t \in (0,T), \tag{4.1}$$

with the initial condition

$$y(0) = y_0, \tag{4.2}$$

that is, y is a solution to the problem $\mathbf{P}(\Phi, B, f + Mu, y_0)$.

From Theorems 3.5 and 3.6, it follows that there exists a unique solution y to the problem $\mathbf{P}(\Phi, B, f + Mu, y_0)$, it belongs to $L^\infty(0,T;V) \cap H^1(0,T;H)$ and satisfies the estimate

$$\begin{aligned} &\max_{t\in[0,T]} |y(t)|^2 + \operatorname*{ess\,sup}_{t\in[0,T]} \|y(t)\|^p + \int_0^T |y'(t)|^2\,dt \\ &\qquad \le C_1\Big(|y_0|^2 + \Phi(y_0) + 2\int_0^T \big[|f(t)|^2 + |Mu(t)|^2\big]\,dt\Big), \end{aligned} \tag{4.3}$$

where C_1 is a positive constant, which depends only on K, L and T.

Let the following condition hold:

$(\mathcal{G})$ $\quad G : C([0,T];H) \to \mathbb{R}$ is a lower semi-continuous and bounded from below functional, i.e.,

$$\inf_{z\in C([0,T];H)} G(z) > -\infty.$$

Let us define the cost functional $J : U \to \mathbb{R}$ by the following rule:

$$J(u) := G(y(u)) + \mu\|u\|_U^2, \quad u \in U, \tag{4.4}$$

where $\mu > 0$ is a constant, $y(u)$ is the solution to the problem $\mathbf{P}(\Phi, B, f+Mu, y_0)$.

The *optimal control problem* is to find $u^* \in U_\partial$ such that

$$J(u^*) = \inf_{u \in U_\partial} J(u). \tag{4.5}$$

Later, this problem will be called as problem (4.5).

The main result of this paper is stated in the following theorem.

Theorem 4.1 (Main theorem). *Let conditions* $(\mathcal{A}_1)$–$(\mathcal{A}_3)$, $(\mathcal{B})$, $(\mathcal{F})$, $(\mathcal{M})$, *and* $(\mathcal{G})$ *hold. Then problem* (4.5) *has a solution.*

The proof of this theorem will be given in the next section.

5. Proof of the main result

Let us prove Theorem 4.1 using the classical direct method of calculus of variations. Since the cost function J is bounded from below, it implies that there exists the sequence $\{u_k\}_{k=1}^\infty \in U_\partial$ such that

$$J(u_k) \underset{k\to\infty}{\longrightarrow} J_* := \inf_{u \in U_\partial} J(u) > -\infty. \tag{5.1}$$

Thus, it means that the sequence $\{J(u_k)\}_{k=1}^\infty$ is bounded. Taking into account (4.4), one can obtain that the sequence $\{u_k\}_{k=1}^\infty$ is bounded in U, i.e.,

$$\|u_k\|_U \le C_2, \quad k \in \mathbb{N}, \tag{5.2}$$

where C_2 is a constant, which does not depend on k.

Since $M : U \to L^2(0,T;H))$ is a linear continuous operator, then $\{Mu_k\}_{k=1}^\infty$ is bounded in $L^2(0,T;H)$, that is,

$$\|Mu_k\|_{L^2(0,T;H)} \le C_3, \quad k \in \mathbb{N}. \tag{5.3}$$

where C_3 is a constant, which does not depend on k.

For each $k \in \mathbb{N}$, denote $y_k := y(u_k)$, i.e., y_k is a solution to the problem $\mathbf{P}(\Phi, B, f + Mu_k, y_0)$. Taking into account conditions $(\mathcal{F})$ and $(\mathcal{M})$, from Theorem 3.6, it follows that for each $k \in \mathbb{N}$, $y_k \in L^\infty(0,T;V) \cap H^1(0,T;H)$, for a.e. $t \in (0,T)$, $y_k(t) \in D(\partial\Phi)$ and

$$y_k'(t) + \partial\Phi(y_k(t)) + B(y_k)(t) \ni f(t) + Mu_k(t) \quad \text{in } H, \tag{5.4}$$

$$y_k(0) = y_0. \tag{5.5}$$

Also, the following estimate holds:

$$\max_{t\in[0,T]} |y_k(t)|^2 + \operatorname*{ess\,sup}_{t\in[0,T]} \|y_k(t)\|^p + \int_0^T |y_k'(t)|^2\,dt$$

$$\leq C_1\Big(|y_0|^2 + \Phi(y_0) + 2\int_0^T \big(|f(t)|^2 + |Mu_k(t)|^2\big)\,dt\Big), \tag{5.6}$$

where C_1 is a positive constant, which depends only on K, L and T.

Moreover, from the proof of Theorem 3.6, it follows that there exists the sequence $g_k \in L^2(0,T;H)$ such that, for a.e. $t \in (0,T)$, $g_k(t) \in \partial\Phi\big(y_k(t)\big)$ and

$$y_k'(t) + g_k(t) + B(y_k)(t) = f(t) + Mu_k(t) \quad \text{in } H. \tag{5.7}$$

From (5.3), (5.6) and $(\mathcal{F})$, it follows that

$$\{y_k\}_{k=1}^{\infty} \quad \text{is bounded in } L^\infty(0,T;V) \cap C([0,T];H), \tag{5.8}$$

$$\{y_k'\}_{k=1}^{\infty} \quad \text{is bounded in } L^2(0,T;H). \tag{5.9}$$

Taking into account (3.2) and (5.8), we obtain that

$$\{B(y_k)\}_{k=1}^{\infty} \quad \text{is bounded in } L^2(0,T;H). \tag{5.10}$$

From (5.7), taking into account (5.3), (5.9), (5.10), and $(\mathcal{F})$, we obtain

$$\{g_k\}_{k=1}^{\infty} \quad \text{is bounded in } L^2(0,T;H). \tag{5.11}$$

Let us recall that the spaces V and H are reflexive. So, from (5.2), (5.8), (5.9), (5.11), it follows that there exists a subsequence of a sequence $\{(u_k, y_k, g_k)\}_{k=1}^{\infty}$ (which we denote by $\{(u_k, y_k, g_k)\}_{k=1}^{\infty}$) and the functions $u_* \in U_\partial$, $y \in L^\infty(0,T;V) \cap H^1(0,T;H)$ (then $y \in C([0,T];H)$), and $g \in L^2(0,T;H)$ such that

$$u_k \underset{k\to\infty}{\longrightarrow} u_* \qquad \text{weakly} \qquad \text{in } U, \tag{5.12}$$

$$y_k \underset{k\to\infty}{\longrightarrow} y \qquad *\text{-weakly} \qquad \text{in } L^\infty(0,T;V), \tag{5.13}$$

$$y_k \underset{k\to\infty}{\longrightarrow} y \qquad \text{weakly} \qquad \text{in } H^1(0,T;H), \tag{5.14}$$

$$g_k \underset{k\to\infty}{\longrightarrow} g \qquad \text{weakly} \qquad \text{in } L^2(0,T;H). \tag{5.15}$$

Taking into account Proposition 2.2, it can be assumed that

$$y_k \underset{k\to\infty}{\longrightarrow} y \quad \text{in } C([0,T];H). \tag{5.16}$$

Based on $(\mathcal{B})$ and (5.16), we obtain

$$\operatorname*{ess\,sup}_{t\in[0,T]} |B(y_k)(t) - B(y)(t)| \leq L\int_0^T |y_k(s) - y(s)|\,ds \underset{k\to\infty}{\longrightarrow} 0,$$

that is,

$$B(y_k) \underset{k\to\infty}{\longrightarrow} B(y) \quad \text{strongly in } L^\infty(0,T;H). \tag{5.17}$$

From $(\mathcal{M})$ and (5.12), it follows that

$$Mu_k \underset{k\to\infty}{\longrightarrow} Mu_* \quad \text{weakly in } L^2(0,T;H). \tag{5.18}$$

Let $v \in H, \varphi \in C([0,T])$ be arbitrary functions. For a.e. $t \in (0,T)$, we multiply equality (5.7) by v and then we multiply the obtained equality by $\varphi(t)$ and integrate it with respect to $t \in [0,T]$. As a result, we obtain

$$\int_0^T (y_k'(t), v\varphi(t))\,dt + \int_0^T (g_k(t), v\varphi(t))\,dt + \int_0^T \big(B(y_k)(t), v\varphi(t)\big)\,dt \\ = \int_0^T (f(t) + Mu_k(t), v\varphi(t))\,dt, \quad k \in \mathbb{N}. \tag{5.19}$$

Taking into account (5.12)–(5.18), we pass to the limit in (5.19) as $k \to \infty$. As a result, since $v \in H, \varphi \in C([0,T])$ are arbitrary, for a.e. $t \in (0,T)$, we obtain

$$y'(t) + g(t) + B(y)(t) = f(t) + (Mu_*)(t) \quad \text{in } H.$$

In order to complete the proof of the theorem, it remains to show that $y(t) \in D(\partial\Phi)$ and $g(t) \in \partial\Phi\big(y(t)\big)$ for a.e. $t \in (0,T)$.

Let $k \in \mathbb{N}$ be an arbitrary number. Since $y_k(t) \in D(\partial\Phi)$ and $g_k(t) \in \partial\Phi\big(y_k(t)\big)$ for a.e. $t \in (0,T)$, we obtain, using the monotonicity of the subdifferential $\partial\Phi$, that, for a.e. $t \in (0,T)$, the following inequality holds:

$$(g_k(t) - v^*, y_k(t) - v) \ge 0, \quad [v, v^*] \in \partial\Phi. \tag{5.20}$$

Let $\sigma \in (0,T)$, $h > 0$ be arbitrary numbers such that $\sigma - h \in (0,T)$. We integrate (5.20) on $(\sigma - h, \sigma)$ and obtain

$$\int_{\sigma-h}^{\sigma} (g_k(t) - v^*, y_k(t) - v)\,dt \ge 0, \quad [v, v^*] \in \partial\Phi. \tag{5.21}$$

Now, according to (5.15) and (5.16), we pass to the limit in (5.21) as $k \to \infty$. As a result, we obtain

$$\int_{\sigma-h}^{\sigma} (g(t) - v^*, y(t) - v)\,dt \ge 0, \quad [v, v^*] \in \partial\Phi. \tag{5.22}$$

Theorem 2 of the monograph [27, p. 192] and (5.22) imply that for every $[v, v^*] \in \partial\Phi$ there exists a set $R_{[v,v_*]} \subset (0,T)$ of measure zero such that for all $\sigma \in (0,T) \setminus R_{[v,v_*]}$ we have $y(\sigma) \in V,\ g(\sigma) \in H$ and

$$0 \le \lim_{h\to+0} \frac{1}{h} \int_{\sigma-h}^{\sigma} \big(g(t) - v^*, y(t) - v\big)\,dt = \big(g(\sigma) - v^*, y(\sigma) - v\big). \tag{5.23}$$

Let us show that there exists a set of measure zero $R \subset (0,T)$ such that $\forall \sigma \in (0,T) \setminus R$:

$$\big(g(\sigma) - v^*, y(\sigma) - v\big) \ge 0, \quad [v, v^*] \in \partial\Phi. \tag{5.24}$$

Since V and V' are separable spaces, there exists a countable set $F \subset \partial\Phi$ which is dense in $\partial\Phi$. Let us denote $R := \bigcup_{[v,v^*]\in F} R_{[v,v_*]}$. Since the set F is countable, and any countable union of sets of measure zero is a set of measure zero, then R is a set of measure zero. Therefore, for any $\sigma \in (0,T) \setminus R$, the inequality

$$\big(g(\sigma) - v^*, y(\sigma) - v\big) \geq 0, \quad [v, v^*] \in F$$

holds. Let $[\widehat{v}, \widehat{v}^*]$ be an arbitrary element from $\partial\Phi$. Then, since F is dense in $\partial\Phi$, we have the existence of a sequence $\{[v_l, v_l^*]\}_{l=1}^{\infty}$ such that $v_l \to \widehat{v}$ in V, $v_l^* \to \widehat{v}^*$ in V'. From the above, it follows that for all $\sigma \in (0,T) \setminus R$ we have

$$(g(\sigma) - v_l^*, y(\sigma) - v_l) \geq 0, \quad l \in \mathbb{N}. \tag{5.25}$$

Thus, passing to the limit in inequality (5.25) as $l \to \infty$, we get $(g(\sigma) - \widehat{v}^*, y(\sigma) - \widehat{v}) \geq 0$ for all $\sigma \in (0,T) \setminus R$. Therefore, inequality (5.24) holds. From this, according to the maximal monotonicity of $\partial\Phi$, we obtain that $[y(t), g(t)] \in \partial\Phi$ for a.e. $t \in (0,T)$, that is, $y(t) \in D(\partial\Phi)$ and $g(t) \in \partial\Phi(y(t))$ for a.e. $t \in (0,T)$. Thus, the function y is a solution to the problem $\mathbf{P}(\Phi, B, f + Mu_*, y_0)$, i.e., $y = y(u_*)$ is the state of the control evolutionary system for a given control u_*.

It remains to show that u_* is a minimizing element of the functional J. Indeed, since the functional G is lower semi-continuous in $C([0,T];H)$, then (5.16) implies that

$$\varliminf_{k\to\infty} G(y(u_k)) \geq G(y(u_*)). \tag{5.26}$$

Also, (5.12) and Proposition 2.1 yield

$$\varliminf_{k\to\infty} \|u_k\|_U \geq \|u_*\|_U. \tag{5.27}$$

From (4.4), (5.1), (5.26), and (5.27), we obtain that

$$\begin{aligned}\inf_{u\in U_\partial} J(u) = \lim_{k\to\infty} J(u_k) &\geq \varliminf_{k\to\infty} G(y(u_k)) + \mu \varliminf_{k\to\infty} \|u_k\|_U^2 \\ &\geq G(y(u_*)) + \mu\|u_*\|_U^2 = J(u_*).\end{aligned}$$

Thus, we obtain that u_* is a solution to problem (4.5). Hence, Theorem 4.1 is proved.

6. Comments on the main result

Let us introduce an example of the problem which is studied here. Let $n \in \mathbb{N}$ be a given number, Ω be a bounded domain in $\mathbb{R}^n$ with the boundary $\partial\Omega$. We put $Q := \Omega \times (0,T)$, $\Sigma := \partial\Omega \times (0,T)$, $\Pi := \{(t,s) \mid 0 \leq s \leq t \leq T\}$.

Let $L^p(\Omega)$, $L^p(Q)$ be standard Lebesgue spaces. We denote by $W^{1,p}(\Omega) = \{v \in L^p(\Omega) \mid v_{x_i} \in L^p(\Omega),\ i = \overline{1,n}\}$, a standard Sobolev space with the norm $\|v\|_{W^{1,p}(\Omega)} := \left(\sum\limits_{i=1}^{n} \|v_{x_i}\|_{L^p(\Omega)}^p + \|v\|_{L^p(\Omega)}^p\right)^{1/p}$.

We consider the operator $B : C([0,T];L^2(\Omega)) \to L^\infty(0,T;L^2(\Omega))$, defined by the following rule: for each function $z(x,t)$, $(x,t) \in Q$, from $C([0,T];L^2(\Omega))$, we have

$$B(z)(x,t) := \int_0^t b(x,t,s)\, z(x,s)\, ds, \quad (x,t) \in Q,$$

where $b \in L^\infty(\Omega \times \Pi)$ is given. Notice that for a.e. $t \in (0,T)$ and every $z \in C([0,T]; L^2(\Omega))$ we obtain

$$\|B(z)(\cdot,t)\|_{L^2(\Omega)} \le \int_0^t \|b(\cdot,t,s)\, z(\cdot,s)\|_{L^2(\Omega)}\, ds \le L \int_0^t \|z(\cdot,s)\|_{L^2(\Omega)}\, ds, \quad (6.1)$$

where $L := \operatorname*{ess\,sup}_{(x,t,s)\in\Omega\times\Pi} |b(x,t,s)|$. So, B is a Volterra type operator.

Let $U := L^2(Q)$ be a space of controls, and U_∂ be a convex closed subset of U, the set of admissible controls. For example, $U_\partial := \{u \in U \mid m \le u \le M$ a.e. on $Q\}$ for given $m, M \in \mathbb{R}$. Also, let K be a convex closed set in $W^{1,p}(\Omega)$ which contains 0. For example, we set $K = \{v \in W^{1,p}(\Omega) \mid v \ge 0$ almost every on $\Omega\}$.

For a given control $u \in U_\partial$, the state of evolutionary system is described by a function $y(x,t)$, $(x,t) \in \overline{Q}$, (also, it can be denoted by y, or $y(u)$, or $y(x,t;u)$, $(x,t) \in \overline{Q}$) such that $y \in L^p(Q) \cap C([0,T]; L^2(\Omega))$, $y_{x_i} \in L^p(Q)$, $i = \overline{1,n}$, $y_t \in L^2(Q)$, and

$$y|_{t=0} = y_0(x), \quad x \in \Omega, \quad (6.2)$$

and, for a.e. $t \in (0,T]$, $y(\cdot,t) \in K$ and

$$\int_{\Omega_t} \big[y_t(v-y) + |\nabla y|^{p-2}\nabla y \nabla(v-y) + |y|^{p-2}y(v-y) + B(y)(v-y)\big]\, dx$$
$$\ge \int_{\Omega_t} (f+u)(v-y)\, dx, \quad v \in K, \quad (6.3)$$

where $f \in L^2(Q)$, $y_0 \in L^2(\Omega)$, $u \in U_\partial$ are given, $\Omega_t := \Omega \times \{t\}$, $\nabla y := (y_{x_1}, \dots, y_{x_n})$.

The cost functional $J : U \to \mathbb{R}$ is defined by the following rule:

$$J(u) = \|y(\cdot,T;u) - y_T(\cdot)\|^2_{L^2(\Omega)} + \mu\|u\|^2_{L^2(Q)}, \quad (6.4)$$

where $\mu > 0$, $y_T \in L^2(\Omega)$ are given.

The *optimal control problem* is to find a control $u_* \in U_\partial$ such that

$$J(u_*) = \inf_{u\in U_\partial} J(u). \quad (6.5)$$

We should remark that the formulated above problem can be written in a more abstract way noting that variational inequality (6.3) can be written as a subdifferential inclusion.

Indeed, after an appropriate identification of functions and functionals, one can write

$$W^{1,p}(\Omega) \subset L^2(\Omega) \subset (W^{1,p}(\Omega))',$$

where all embeddings are dense, continuous and compact. Clearly, for any $h \in L^2(\Omega)$ and $v \in W^{1,p}(\Omega)$, we have $\langle h, v\rangle = (h, v)$, where by $\langle\cdot,\cdot\rangle$ we denote the scalar product on the dual pair $\big[(W^{1,p}(\Omega))',\, W^{1,p}(\Omega)\big]$, and by $(\cdot,\cdot)$ we denote the scalar product in $L^2(\Omega)$. Thus, we may use the notation $(\cdot,\cdot)$ instead of $\langle\cdot,\cdot\rangle$.

Now, let us denote $V := W^{1,p}(\Omega)$, $H := L^2(\Omega)$, and define the operator $A : V \to V'$ as follows:

$$(A(v), w) := \int_\Omega \left[|\nabla v|^{p-2}\nabla v \nabla w + |v|^{p-2} v w\right] dx, \quad v, w \in V.$$

We also use the notations:

$$\begin{gathered} B(y)(t) := B(y)(\cdot, t), \quad f(t) := f(\cdot, t), \quad u(t) := u(\cdot, t), \quad t \in (0, T), \\ y(t) := y(\cdot, t), \quad y(t; u) := y(\cdot, t; u), \quad t \in [0, T]. \end{gathered}$$

Then, for a given control $u \in U_\partial \subset L^2(0, T; H)$, the state of evolutionary system is described by a function $y \in L^p(0, T; V)$ such that $y' \in L^2(0, T; H)$, $y(0) = y_0$ and, for a.e. $t \in (0, T)$, $y(t) \in K \subset V$ and

$$\big(y'(t) + A(y(t)) + B(y)(t), v - y(t)\big) \geq \big(f(t) + u(t), v - y(t)\big), \quad v \in K, \tag{6.6}$$

where $f \in L^2(0, T; H)$, $y_0 \in H$ are given.

We remark that, for a.e. $t \in S$, variational inequality (6.6) can be written as

$$(y'(t) + A(y(t)) + B(y)(t) - f(t) - u(t), v - y(t)) + I_K(v) - I_K(y(t)) \geq 0, \quad v \in V, \tag{6.7}$$

where

$$I_K(v) := \begin{cases} 0 & \text{if } v \in K \\ +\infty & \text{if } v \in V \setminus K \end{cases}.$$

We can write inequality (6.7) as follows:

$$I_K(v) \geq I_K(y(t)) + (-y'(t) - A(y(t)) - B(y)(t) + f(t) + u(t), v - y(t)), \quad v \in V. \tag{6.8}$$

By the definition of the subdifferential ∂I_K, inequality (6.8) is equivalent to the inclusion

$$\begin{aligned} &\partial I_K(y(t)) \ni -y'(t) - A(y(t)) - B(y)(t) + f(t) + u(t) \\ &\qquad \Leftrightarrow y'(t) + A(y(t)) + \partial I_K(y(t)) + B(y)(t) \ni f(t) + u(t). \end{aligned}$$

We define

$$\Psi(v) := \frac{1}{p}\int_\Omega \left[|\nabla v|^p + |v|^p\right] dx, \quad \Phi(v) := \Psi(v) + I_K(v), \quad v \in V.$$

The functionals Ψ, Φ from V to $\mathbb{R}_\infty$ are proper, convex and semi-lower-continuous. Let $\partial\Psi, \partial\Phi : V \to 2^{V'}$ be the subdifferentials of Ψ, Φ. We have $\partial\Psi(v) = \{A(v)\} \subset V'$ for each $v \in V$, and $\partial\Phi(v) := A(v) + \partial I_K(v)$, $v \in V$.

From the above, it follows that finding of a solution to variational inequality (6.6) with the initial condition $y(0) = y_0$ is equivalent to finding of *a function $y \in L^p(0, T; V) \cap C([0, T]; H)$ such that $y' \in L^2(0, T; H)$, $y(0) = y_0$ and, for a.e. $t \in (0, T)$, $y(t) \in D(\partial\Phi)$ and*

$$y'(t) + \partial\Phi(y(t)) + B(y)(t) \ni f(t) + u(t) \quad \textit{in } H.$$

In this case (see (6.4)), the cost functional $J : U \to \mathbb{R}$ is defined by

$$J(u) := \|y(T; u) - y_T\|_H^2 + \mu\|u\|_U^2,$$

where $y_T \in H$ is given.

It follows from Theorem 4.1 that there is a solution to the stated optimal control problem (6.5).

Remark 6.1. Other than the above in this section, the operator B is a defined operator

$$B(w)(t) := \widetilde{B}\left(t, \int_0^t \widetilde{b}(t, s, w(s))\, ds\right), \quad t \in (0, T), \tag{6.9}$$

where $\widetilde{B} : (0, T) \times H \to H$, $\widetilde{b} : \Pi \times H \to H$ are maps which satisfy the following conditions:

1) for any $v \in H$, the map $\widetilde{B}(\cdot, v) : (0, T) \to H$ is measurable, and there exists a constant $L_1 \geq 0$ such that the inequality

$$|\widetilde{B}(t, v_1) - \widetilde{B}(t, v_2)| \leq L_1|v_1 - v_2| \tag{6.10}$$

holds for a.e. $t \in (0, T)$ and for all $v_1, v_2 \in H$; in addition, $\widetilde{B}(t, 0) = 0$ for a.e. $t \in (0, T)$;

2) for any $v \in H$, the map $\widetilde{b}(\cdot, \cdot, v) : \Pi \to H$ is measurable, and there exists a constant $L_2 \geq 0$ such that the inequality

$$|\,\widetilde{b}(t, s, v_1) - \widetilde{b}(t, s, v_2)| \leq L_2|v_1 - v_2| \tag{6.11}$$

holds for a.e. $(t, s) \in \Pi$ and for all $v_1, v_2 \in H$; in addition, $\widetilde{b}(t, s, 0) = 0$ for a.e. $(t, s) \in \Pi$.

Let us show that the operator B, defined in (6.9), satisfies condition $(\mathcal{A}_4)$ with $L = L_1L_2$. Indeed, due to (6.10) and (6.11), we have

$$\begin{aligned}|B(w_1)(t) - B(w_2)(t)| &\leq \big|\widetilde{B}\big(t, \int_0^t \widetilde{b}(t, s, w_1(s))\, ds\big) - \widetilde{B}\big(t, \int_0^t \widetilde{b}(t, s, w_2(s))\, ds\big)\big| \\ &\leq L_1\Big|\int_0^t \widetilde{b}(t, s, w_1(s))\, ds - \int_0^t \widetilde{b}(t, s, w_2(s))\, ds\Big| \leq L_1L_2 \int_0^t |w_1(s) - w_2(s)|\, ds.\end{aligned}$$

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Mykola Bokalo,
Ivan Franko National University of Lviv, Department of Mathematical Statistics and Differential Equations, 1 Universitetska Street, Lviv, 79000, Ukraine,
E-mail: mm.bokalo@gmail.com

Olha Sus,
Tufts University, Department of Education, 12 Upper Campus Road, Medford, 02155, MA, USA,
E-mail: olha.sus@tufts.edu

Задачі оптимального керування для еволюційних варіаційних нерівностей з операторами типу Вольтерра

Mykola Bokalo and Olha Sus

У цій статті розглядаємо задачу оптимального керування для класу еволюційних субдиференціальних включень з операторами типу Вольтерра. Результати стосовно існування та єдиності розв'язку задачі з початковою умовою для таких включень були отримані в попередній нашій роботі. Тут встановлюємо існування розв'язку поставленої задачі оптимального керування за деяких припущень на вхідні дані. Мотивацією для цієї роботи є задачі оптимального керування системами, що описуються еволюційними варіаційними нерівностями, що виникають при вивченні процесів лиття під тиском, контактної механіки, процесів електрозмочування діелектрика та інших.

Ключові слова: параболічна варіаційна нерівність, еволюційна варіаційна нерівність, оператор типу Вольтерра, оптимальне керування