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The Interaction of a Countable Number of Eddy Flows for the Bryan–Pidduck Model

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We consider the nonlinear integro-differential Boltzmann equation for the model of rough spheres. An approximate solution is constructed in the form of a linear combination of a countable number of the Maxwell modes with certain coefficient functions that depend on time and spatial coordinate. Sufficient conditions for an arbitrarily small uniform-integral error are obtained.

Key words: Bryan–Pidduck equation, rough spheres, eddy flows, infinite modal distribution

Mathematical Subject Classification 2020: 76P05, 45K05, 82C40, 35Q20

1. Introduction

We consider a model of rough spheres [2] which was firstly introduced in 1894 by Bryan [1]. The methods developed by Chapman and Enskog for general non-rotating spherical molecules were extended to Bryan's model by Pidduck [13] in 1922. The advantage of this model over all other models that allow a change in the rotational state of the molecule is that it does not require any additional variables that determine the orientation of the molecule in space.

These molecules are absolutely elastic and absolutely rough, which means the following. When two molecules collide, the points in contact do not generally have the same velocity. The two spheres are assumed to engage each other without slipping. At the initial moment, the spheres deform each other, and after the collision, the deformation energy returns to the kinetic energy of translational and rotational motion without any loss. As a result, the relative velocity of the spheres at the point of contact is reversed by the impact.

The Boltzmann equation for the model of rough spheres (or the Bryan–Pidduck equation) has the form [1–3]:

$$D(f) = Q(f, f), \tag{1.1}$$

where the differential operator $D(f)$ is defined as follows:

$$D(f) \equiv \frac{\partial f}{\partial t} + \left(V, \frac{\partial f}{\partial x}\right), \tag{1.2}$$

and the collision integral $Q(f, f)$ for the Bryan–Pidduck model has the form

$$Q(f, f) \equiv \frac{d^2}{2} \int_{R^3} dV_1 \int_{R^3} d\omega_1 \int_{\Sigma} d\alpha \, B(V - V_1, \alpha)$$

$$\times \Big[f(t, V_1^*, x, \omega_1^*) f(t, V^*, x, \omega^*) - f(t, V, x, \omega) f(t, V_1, x, \omega_1) \Big]. \tag{1.3}$$

Here, d is the diameter of the molecule, which is associated with the moment of inertia I by the relation

$$I = \frac{bd^2}{4}, \tag{1.4}$$

where b is the parameter, $b \in \left(0, \frac{2}{3}\right]$, characterizing the isotropic distribution of matter inside the gas particles; t is the time; $x = (x^1, x^2, x^3) \in \mathbb{R}^3$ is the spatial coordinate; $V = (V^1, V^2, V^3) \in \mathbb{R}^3$ and $\omega = (\omega^1, \omega^2, \omega^3) \in \mathbb{R}^3$ denote the linear and angular velocities of the molecule, respectively; $\frac{\partial f}{\partial x}$ is the gradient of the desired distribution function by variable x; the unit sphere in the space $\mathbb{R}^3$ is denoted by Σ; $\alpha \in \mathbb{R}^3$ is the unit vector directed along the line connecting the centers of the colliding molecules; (a, b) is the dot product of the vectors a and b; the collision term at the integral $Q(f, f)$ (1.3) is defined as follows:

$$B\left(V - V_1, \alpha\right) = |(V - V_1, \alpha)| - (V - V_1, \alpha). \tag{1.5}$$

The linear V^*, V_1^* and angular ω^*, ω_1^* molecular velocities after the collision can be expressed by the appropriate values before the collision as follows from [2]:

$$\begin{aligned}
V^* &= V - \frac{1}{b+1}\left(b(V_1 - V) - \frac{bd}{2}\left[\alpha, \omega + \omega_1\right] + \alpha(\alpha, V_1 - V) \right), \\
V_1^* &= V_1 + \frac{1}{b+1}\left(b(V_1 - V) - \frac{bd}{2}\left[\alpha, \omega + \omega_1\right] + \alpha(\alpha, V_1 - V) \right), \\
\omega^* &= \omega + \frac{2}{d(b+1)}\left\{ [\alpha, V - V_1] + \frac{d}{2}\left(\alpha(\omega + \omega_1, \alpha) - \omega - \omega_1\right) \right\}, \\
\omega_1^* &= \omega_1 + \frac{2}{d(b+1)}\left\{ [\alpha, V - V_1] + \frac{d}{2}\left(\alpha(\omega + \omega_1, \alpha) - \omega - \omega_1\right) \right\},
\end{aligned} \tag{1.6}$$

where $[a, b]$ indicates the vector product of the vectors a and b.

2. Statement of the problem

The only exact solution to equation (1.1), which is explicitly known up to now, is an expression similar to that obtained by Maxwell [2] for the model of hard spheres, commonly called Maxwellian [2], which makes both parts of the Boltzmann equation (1.2) and (1.3) equal to zero.

The most general form of the local Maxwellian, which exists for the rough sphere model, was obtained in [10]. Since any other exact solutions have not yet been found in an explicit form, it is of interest to look for an explicit form of approximate solutions. One of the variants of this form is a linear combination of Maxwellians.

As the Maxwell distributions, we consider the eddy flow [10]:

$$M_i(V, \omega, t, x) = \rho_i I^{3/2} \left(\frac{\beta_i}{\pi}\right)^3 e^{-\beta_i\left(\left(V - \overline{V}_i\right)^2 + I\omega^2\right)}, \tag{2.1}$$

(by default, here and everywhere below the index i belongs to the set of natural numbers) where the density of a gas flow has the form

$$\rho_i = \rho_{0i} e^{\beta_i \overline{\omega}_i^2 r_i^2}. \tag{2.2}$$

By ρ_{0i}, we denote the gas density on the axis of rotation, the parameter β_i is the inverse temperature

$$\beta_i = \frac{1}{2T_i}; \tag{2.3}$$

$\overline{\omega}_i$ is the angular velocity of i-th gas flow around the axis x_{0i}, which at the initial moment of time $t = 0$ passes through the point

$$x_{0i} = \frac{1}{\overline{\omega}_i^2} \left[\overline{\omega}_i, \widetilde{V}_i \right]; \tag{2.4}$$

the arbitrary constant vector $\widetilde{V}_i$ belongs to the space $\mathbb{R}^3$. The density axis is the line around which the gas density of the flow is symmetrically distributed (in detail [8]), and one of the $i-$th flows passes through the point

$$\overline{x}_{0i} = \frac{1}{\overline{\omega}_i^2} \left[\overline{\omega}_i, \widetilde{V}_i - \overline{u}_{0i} \right] \tag{2.5}$$

at the moment of time $t = 0$.

The distance between the molecule and the axis of rotation $\overline{x}_{0i}$ has the form

$$r_i^2 = \frac{1}{\overline{\omega}_i^2} \left[\overline{\omega}_i, x - \overline{x}_{0i} - \overline{u}_{0i} t \right]^2. \tag{2.6}$$

If we denote by $\overline{u}_{0i}$ the linear velocity of the axis x_{0i} around which the flow rotates, then

$$\overline{u}_{0i} \perp \overline{\omega}_i. \tag{2.7}$$

The mass velocity of a gas flow $\overline{V}_i$ has the form

$$\overline{V}_i = \widehat{V}_i + \left[\overline{\omega}_i, x - x_{0i} - \overline{u}_{0i} t \right], \tag{2.8}$$

where $\widehat{V}_i$ is the linear velocity of the gas flow along the axis of rotation, i.e.,

$$\widehat{V}_i \parallel \overline{\omega}_i. \tag{2.9}$$

From a physical point of view, the expression (2.1) describes a gas flow that rotates around the x_{0i} axis, simultaneously moves along it and flies in a direction perpendicular to this axis. With this gas movement it forms two axes: the velocity axis x_{0i} and the density axis $\overline{x}_{0i}$. On the different sides of the axis $\overline{x}_{0i}$ the rarefaction and compression zones appear.

In [9], a bimodal distribution with eddy-like Maxwell modes was constructed. In [11], the infinite-modal solution of the Bryan–Pidduck equation was given. There we considered two cases of Maxwellians: the global Maxwellian and one of the local Maxwellians, namely, the screw. Now we would like to continue [11] and to construct an approximate solution of equation (1.1)–(1.3) in the following form:

$$f(V,\omega,t,x)=\sum_{i=1}^{\infty}\varphi_i(t,x)M_i(V,\omega,t,x), \tag{2.10}$$

namely, a countable modal linear combination of some coefficient functions $\varphi_i(t,x)$ and Maxwellians. As the coefficient functions $\varphi_i(t,x)$ of the distribution (2.10), we consider some nonnegative smooth functions defined in time and space. We demand that the norm

$$\|h(t,x)\|=\sup_{(t,x)\in\mathbb{R}^4}\left(|h(t,x)|+\left|\frac{\partial h(t,x)}{\partial t}\right|+\left|\frac{\partial h(t,x)}{\partial x}\right|\right) \tag{2.11}$$

be not equal to zero for all functions $\varphi_i(t,x)$.

To estimate the approximation of the solution, we use the uniform-integral (or mixed) error between the parts of equation (1.1), which was first proposed in [7],

$$\Delta=\Delta(\{\beta_i\}_{i=1}^{\infty})=\sup_{(t,x)\in\mathbb{R}^4}\int_{\mathbb{R}^3}dV\int_{\mathbb{R}^3}d\omega\,\bigl|D(f)-Q(f,f)\bigr|. \tag{2.12}$$

The aim of this paper is to find the form of the coefficient functions $\varphi_i(t,x)$ and the hydrodynamic parameters of Maxwellian (2.1) such that error (2.12) could be made arbitrarily small.

3. Main results

First, we give several statements that contain the main result of the present work.

Theorem 3.1. *Let the coefficient functions $\varphi_i(t,x)$ in the distribution (2.10) have the form*

$$\varphi_i(t,x)=e^{-\beta_i\overline{\omega}_i^2 r_i^2}\psi_i(t,x), \tag{3.1}$$

where $\psi_i(t,x)$ are nonnegative smooth functions with nonzero norm (2.11). We also require that the functional series with common terms have one of the following forms:

$$\psi_i,\quad |x|\psi_i,\quad t\psi_i,\quad \left|\frac{\partial\psi_i}{\partial x}\right|,\quad \left|\frac{\partial\psi_i}{\partial t}\right|,\quad |x|\left|\frac{\partial\psi_i}{\partial x}\right|,\quad t\left|\frac{\partial\psi_i}{\partial t}\right|, \tag{3.2}$$

which converges uniformly on $\mathbb{R}^4$. Also let

$$\overline{\omega}_i=\overline{\omega}_{0i}\beta_i^{-m_i},\quad m_i\geqslant\frac{1}{4}. \tag{3.3}$$

Then there exists a function $\Delta'(\{\beta_i\}_{i=1}^{\infty})$ such that

$$\Delta\leqslant\Delta', \tag{3.4}$$

and one of the following forms of the low-temperature limit is true:

a) *if* $m_i > \frac{1}{2}$, *then*

$$L := \lim_{\beta_i \to +\infty} \Delta' = \sum_{i=1}^{\infty} \rho_{0i} \sup_{(t,x)\in\mathbb{R}^4} \left| \frac{\partial \psi_i}{\partial t} + \left(VW_i, \frac{\partial \psi_i}{\partial x} \right) \right| + 2\pi d^2 \sum_{\substack{i,j=1\\ i\neq j}}^{\infty} \rho_{0i}\rho_{0j} \left| VW_i - VW_j \right| \sup_{(t,x)\in\mathbb{R}^4} \psi_i(t,x)\psi_j(t,x), \tag{3.5}$$

where

$$VW_i = \widehat{V}_i + \widetilde{V}_i - \frac{1}{\overline{\omega}_{0i}^2} \overline{\omega}_{0i} \left(\overline{\omega}_{0i}, \widetilde{V}_i \right); \tag{3.6}$$

b) *if* $m_i = \frac{1}{2}$, *then*

$$\lim_{\beta_i \to +\infty} \Delta' = L + 2 \sum_{i=1}^{\infty} \rho_{0i} \left| \left[\overline{\omega}_{0i}, \widetilde{V}_i - \overline{u}_{0i} \right] \right| \sup_{(t,x)\in\mathbb{R}^4} \psi_i(t,x); \tag{3.7}$$

c) *for* $m_i \in \left(\frac{1}{4}, \frac{1}{2}\right)$, *we require an additional parallel condition*

$$\overline{\omega}_{0i} \parallel \left(\widetilde{V}_i - \overline{u}_{0i} \right) \tag{3.8}$$

for the low-temperature limit (3.5) *to remain true;*

d) *for the cutoff value* $m_i = \frac{1}{4}$, *we change the condition* (3.3) *as follows:*

$$\overline{\omega}_i = \overline{\omega}_{0i} s_i \beta_i^{-\frac{1}{4}}, \tag{3.9}$$

where s_i *are positive constants, and we add the condition* (3.8) *to obtain the low-temperature limit*

$$\lim_{\beta_i \to +\infty} \Delta' = L + \frac{4}{\sqrt{\pi}} \sum_{i=1}^{\infty} \rho_{0i} s_i^2 \overline{\omega}_{0i}^2 \sup_{(t,x)\in\mathbb{R}^4} \left(2|x| + |\overline{u}_{0i}|\, t \right) \psi_i(t,x). \tag{3.10}$$

Remark 3.2. The notation $\beta_i \to +\infty$ means that for any value i there exists such a positive $\overline{\beta}$ that the inequality $\beta_i > \overline{\beta}$ is true, moreover $\overline{\beta} \to +\infty$.

Proof of Theorem 3.1. The following inequality was obtained in [11]:

$$\int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega \left| D(f) - Q(f,f) \right| \leqslant \sum_{i=1}^{\infty} \int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega M_i \left| \frac{\partial \varphi_i}{\partial t} + \left(V, \frac{\partial \varphi_i}{\partial x} \right) \right| + 2 \sum_{\substack{i,j=1\\ i\neq j}}^{\infty} \frac{d^2 \rho_i \rho_j}{\pi^2} \varphi_i \varphi_j \int_{\mathbb{R}^3} dq \int_{\mathbb{R}^3} dq_1\, e^{-q^2-q_1^2} \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \overline{V}_i - \overline{V}_j \right|. \tag{3.11}$$

For further calculations, we will need the vector relation that holds for any four vectors on $\mathbb{R}^3$:

$$\left([a,b], [c,d] \right) = (a,c)(b,d) - (a,d)(b,c). \tag{3.12}$$

To differentiate the function $\varphi_i(t,x)$ with respect to the time t, we use (3.1),

$$\begin{aligned}\frac{\partial\varphi_i}{\partial t} &= e^{-\beta_i\overline{\omega}_i^2 r_i^2}\frac{\partial\psi_i}{\partial t} + \psi_i e^{-\beta_i\overline{\omega}_i^2 r_i^2}\frac{\partial}{\partial t}\left(-\beta_i\overline{\omega}_i^2 r_i^2\right)\\ &= e^{-\beta_i\overline{\omega}_i^2 r_i^2}\left\{\frac{\partial\psi_i}{\partial t} - \beta_i\psi_i\left(2\overline{u}_{0i}^2\overline{\omega}_i^2 t - 2\overline{\omega}_i^2\left(x-\overline{x}_{0i},\overline{u}_{0i}\right)\right)\right\}.\end{aligned}\tag{3.13}$$

By using definition (2.5), the derivative takes the form

$$\frac{\partial\varphi_i}{\partial t} = e^{-\beta_i\overline{\omega}_i^2 r_i^2}\left\{\frac{\partial\psi_i}{\partial t} + 2\beta_i\psi_i\overline{\omega}_i^2\left(x,\overline{u}_{0i}\right) - 2\beta_i\psi_i\overline{u}_{0i}^2\overline{\omega}_i^2 t - 2\beta_i\psi_i\left(\left[\overline{\omega}_i,\widetilde{V}_i\right],\overline{u}_{0i}\right)\right\}.\tag{3.14}$$

Next, we differentiate the coefficient function $\varphi_i(t,x)$ with respect to the spatial coordinate x,

$$\begin{aligned}\frac{\partial\varphi_i}{\partial x} &= e^{-\beta_i\overline{\omega}_i^2 r_i^2}\frac{\partial\psi_i}{\partial x} + \psi_i e^{-\beta_i\overline{\omega}_i^2 r_i^2}\left(-\beta_i\frac{\partial}{\partial x}\left(\left[\overline{\omega}_i, x-\overline{x}_{0i}-\overline{u}_{0i}t\right]^2\right)\right)\\ &= e^{-\beta_i\overline{\omega}_i^2 r_i^2}\left\{\frac{\partial\psi_i}{\partial x} + 2\beta_i\psi_i\left(\overline{\omega}_i\left(\overline{\omega}_i,x\right) - \overline{\omega}_i^2\left(x-\overline{x}_{0i}-\overline{u}_{0i}t\right)\right)\right\}.\end{aligned}\tag{3.15}$$

The derivatives found for the function $\varphi_i(t,x)$, (3.14), (3.15) and the Maxwellian M_i (2.1) allow us to write the estimate

$$\begin{aligned}&\int_{\mathbb{R}^3} dV\int_{\mathbb{R}^3} d\omega\left|D(f)-Q(f,f)\right|\\ &\quad\leqslant\sum_{i=1}^{\infty}\rho_{0i}I^{3/2}\left(\frac{\beta_i}{\pi}\right)^3\int_{\mathbb{R}^3} dV\int_{\mathbb{R}^3} d\omega\, e^{-\beta_i\left(\left(V-\overline{V}_i\right)^2+I\omega^2\right)}\\ &\quad\times\left|\frac{\partial\psi_i}{\partial t} + \left(V,\frac{\partial\psi_i}{\partial x}\right) + 2\beta_i\psi_i\left(\overline{\omega}_i^2\left(x,\overline{u}_{0i}\right) - \overline{u}_{0i}^2\overline{\omega}_i^2 t - \left(\left[\overline{\omega}_i,\widetilde{V}_i\right],\overline{u}_{0i}\right)\right)\right.\\ &\quad\left. + 2\beta_i\psi_i\left(V,\overline{\omega}_i\left(\overline{\omega}_i,x\right) - \overline{\omega}_i^2\left(x-\overline{x}_{0i}-\overline{u}_{0i}t\right)\right)\right| + S,\end{aligned}\tag{3.16}$$

where S is as follows:

$$S := \frac{2d^2}{\pi^2}\sum_{\substack{i,j=1\\ i\neq j}}^{\infty}\rho_{0i}\rho_{0j}\psi_i\psi_j\int_{\mathbb{R}^3} dq\int_{\mathbb{R}^3} dq_1\, e^{-q^2-q_1^2}\left|\frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \overline{V}_i - \overline{V}_j\right|.\tag{3.17}$$

Let us calculate the integral over the angular velocity ω of the gas molecule and over the linear velocity V perform the substitution

$$V = \frac{p}{\sqrt{\beta_i}} + \overline{V}_i,\tag{3.18}$$

whose Jacobian J is equal to $\beta_i^{-3/2}$. Then, starting from (3.16), we obtain the inequality

$$\int_{\mathbb{R}^3} dV\int_{\mathbb{R}^3} d\omega\left|D(f)-Q(f,f)\right|$$

$$\leqslant \sum_{i=1}^{\infty} \rho_{0i}\pi^{-3/2} \int_{\mathbb{R}^3} dp e^{-p^2} \left| \frac{\partial \psi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \overline{V}_i, \frac{\partial \psi_i}{\partial x} \right) \right.$$
$$+ 2\beta_i \psi_i \left(\overline{\omega}_i^2 (x, \overline{u}_{0i}) - \overline{u}_{0i}^2 \overline{\omega}_i^2 t - \left(\left[\overline{\omega}_i, \widetilde{V}_i \right], \overline{u}_{0i} \right) \right)$$
$$\left. + 2\beta_i \psi_i \left(\frac{p}{\sqrt{\beta_i}} + \overline{V}_i, \overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 (x - \overline{x}_{0i} - \overline{u}_{0i} t) \right) \right| + S. \quad (3.19)$$

Using (2.8) of the mass velocity $\overline{V}_i$, we can write the last inequality as follows:

$$\int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega \left| D(f) - Q(f,f) \right|$$
$$\leqslant \sum_{i=1}^{\infty} \rho_{0i}\pi^{-3/2} \int_{\mathbb{R}^3} dp e^{-p^2} \left| \frac{\partial \psi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + [\overline{\omega}_i, x - x_{0i} - \overline{u}_{0i} t], \frac{\partial \psi_i}{\partial x} \right) \right.$$
$$+ 2\beta_i \psi_i \left(\overline{\omega}_i^2 (x, \overline{u}_{0i}) - \overline{u}_{0i}^2 \overline{\omega}_i^2 t - \left(\left[\overline{\omega}_i, \widetilde{V}_i \right], \overline{u}_{0i} \right) \right)$$
$$+ 2\beta_i \psi_i \left(\widehat{V}_i + [\overline{\omega}_i, x - x_{0i} - \overline{u}_{0i} t], \overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 (x - \overline{x}_{0i} - \overline{u}_{0i} t) \right)$$
$$\left. + 2\sqrt{\beta_i} \psi_i \left(p, \overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 (x - \overline{x}_{0i} - \overline{u}_{0i} t) \right) \right| + S. \quad (3.20)$$

Using the previously mentioned identity (3.12), we are ready to make the following statement:

$$\left(\widehat{V}_i + [\overline{\omega}_i, x - x_{0i} - \overline{u}_{0i} t], \overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 (x - \overline{x}_{0i} - \overline{u}_{0i} t) \right)$$
$$= \left(\widehat{V}_i, \overline{\omega}_i \right) (\overline{\omega}_i, x) + \left(\widehat{V}_i, \left[\overline{\omega}_i, \widetilde{V}_i - \overline{u}_{0i} \right] \right) - \overline{\omega}_i^2 \left(\widehat{V}_i, x \right)$$
$$- \overline{\omega}_i^2 \left([\overline{\omega}_i, x - x_{0i} - \overline{u}_{0i} t], x - x_{0i} - \overline{u}_{0i} t + \frac{1}{\overline{\omega}_i^2} [\overline{\omega}_i, \overline{u}_{0i}] \right)$$
$$= \left(\widehat{V}_i, \overline{\omega}_i \right) (\overline{\omega}_i, x) - \overline{\omega}_i^2 \left(\widehat{V}_i, x \right) - \left([\overline{\omega}_i, x - x_{0i} - \overline{u}_{0i} t], [\overline{\omega}_i, \overline{u}_{0i}] \right)$$
$$= \left(\widehat{V}_i, \overline{\omega}_i \right) (\overline{\omega}_i, x) - \overline{\omega}_i^2 \left(\widehat{V}_i, x \right) - \overline{\omega}_i^2 (x - x_{0i} - \overline{u}_{0i} t, \overline{u}_{0i})$$
$$= \left(\widehat{V}_i, \overline{\omega}_i \right) (\overline{\omega}_i, x) - \overline{\omega}_i^2 \left(\widehat{V}_i, x \right) - \overline{\omega}_i^2 (x, \overline{u}_{0i}) + \left(\left[\overline{\omega}_i, \widetilde{V}_i \right], \overline{u}_{0i} \right) + \overline{\omega}_i^2 \overline{u}_{0i}^2 t.$$

In its turn, applying equality (3.12), the mutual location of the vectors (2.9) and another term with the first power β_i in (3.20), we see that the terms with β_i are subtracted in the first sum on the right of inequality (3.20). That is, the following estimate takes place:

$$\int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega \left| D(f) - Q(f,f) \right| \leqslant \sum_{i=1}^{\infty} \rho_{0i}\pi^{-3/2} \int_{\mathbb{R}^3} dp e^{-p^2}$$
$$\times \left| \frac{\partial \psi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + [\overline{\omega}_i, x - \overline{u}_{0i} t] - \frac{1}{\overline{\omega}_i^2} \overline{\omega}_i \left(\overline{\omega}_i, \widetilde{V}_i \right) + \widetilde{V}_i, \frac{\partial \psi_i}{\partial x} \right) \right.$$
$$\left. + 2\sqrt{\beta_i} \psi_i \left(p, \overline{\omega}_i (\overline{\omega}_i, x) - \overline{\omega}_i^2 (x - \overline{u}_{0i} t) + \left[\overline{\omega}_i, \widetilde{V}_i - \overline{u}_{0i} \right] \right) \right| + S. \quad (3.21)$$

Further, in the last inequality, we make a transition to the supremum over time and spatial coordinates, the existence of which is ensured by the condition of uniform convergence of the series with the common term (3.2). That is, there really exists a majorant Δ' such that the estimate (3.4) holds and it has the form

$$\Delta' = \sum_{i=1}^{\infty} \rho_{0i}\pi^{-3/2} \int_{\mathbb{R}^3} dp\, e^{-p^2}$$
$$\times \sup_{(t,x)\in\mathbb{R}^4} \left| \frac{\partial \psi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \widehat{V}_i + [\overline{\omega}_i, x - \overline{u}_{0i}t] - \frac{1}{\overline{\omega}_i^2}\overline{\omega}_i \left(\overline{\omega}_i, \widetilde{V}_i\right) + \widetilde{V}_i, \frac{\partial \psi_i}{\partial x} \right) \right.$$
$$\left. + 2\sqrt{\beta_i}\psi_i \left(p, \overline{\omega}_i \left(\overline{\omega}_i, x\right) - \overline{\omega}_i^2 \left(x - \overline{u}_{0i}t\right) + \left[\overline{\omega}_i, \widetilde{V}_i - \overline{u}_{0i}\right]\right) \right|$$
$$+ \frac{2d^2}{\pi^2} \sum_{\substack{i,j=1\\ i\neq j}}^{\infty} \rho_{0i}\rho_{0j} \sup_{(t,x)\in\mathbb{R}^4} \psi_i\psi_j \int_{\mathbb{R}^3} dq \int_{\mathbb{R}^3} dq_1\, e^{-q^2-q_1^2} \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \overline{V}_i - \overline{V}_j \right|. \tag{3.22}$$

Let us apply the average angular velocity (3.3) to write the new expression for Δ':

$$\Delta' = \sum_{i=1}^{\infty} \rho_{0i}\pi^{-3/2} \int_{\mathbb{R}^3} dp\, e^{-p^2}$$
$$\times \sup_{(t,x)\in\mathbb{R}^4} \left| \frac{\partial \psi_i}{\partial t} + \left(VW_i + \beta_i^{-m_i} [\overline{\omega}_{0i}, x - \overline{u}_{0i}t] + \frac{p}{\sqrt{\beta_i}}, \frac{\partial \psi_i}{\partial x} \right) \right.$$
$$\left. + 2\psi_i\beta_i^{\frac{1}{2}-m_i} \left(p, \overline{\omega}_{0i}\left(\overline{\omega}_{0i}, x\right)\beta_i^{-m_i} - \overline{\omega}_{0i}^2 \left(x - \overline{u}_{0i}t\right)\beta_i^{-m_i} + \left[\overline{\omega}_{0i}, \widetilde{V}_i - \overline{u}_{0i}\right]\right) \right|$$
$$+ \frac{2d^2}{\pi^2} \sum_{\substack{i,j=1\\ i\neq j}}^{\infty} \rho_{0i}\rho_{0j} \sup_{(t,x)\in\mathbb{R}^4} \psi_i\psi_j \int_{\mathbb{R}^3} dq \int_{\mathbb{R}^3} dq_1\, e^{-q^2-q_1^2}$$
$$\times \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \beta_i^{-m_i} [\overline{\omega}_{0i}, x - \overline{u}_{0i}t] - \beta_j^{-m_j} [\overline{\omega}_{0j}, x - \overline{u}_{0j}t] + VW_i - VW_j \right|.$$

The substitution

$$\gamma_i = \frac{1}{\beta_i} \tag{3.23}$$

in the last expression of Δ' leads to the following form of the majorant:

$$\Delta' = \sum_{i=1}^{\infty} \rho_{0i}\pi^{-3/2} \int_{\mathbb{R}^3} dp\, e^{-p^2}$$
$$\times \sup_{(t,x)\in\mathbb{R}^4} \left| \frac{\partial \psi_i}{\partial t} + \left(VW_i + \gamma_i^{m_i} [\overline{\omega}_{0i}, x - \overline{u}_{0i}t] + p\sqrt{\gamma_i}, \frac{\partial \psi_i}{\partial x} \right) \right.$$
$$\left. + 2\psi_i\gamma_i^{m_i-\frac{1}{2}} \left(p, \overline{\omega}_{0i}\left(\overline{\omega}_{0i}, x\right)\gamma_i^{m_i} - \overline{\omega}_{0i}^2 \left(x - \overline{u}_{0i}t\right)\gamma_i^{m_i} + \left[\overline{\omega}_{0i}, \widetilde{V}_i - \overline{u}_{0i}\right]\right) \right|$$
$$+ \frac{2d^2}{\pi^2} \sum_{\substack{i,j=1\\ i\neq j}}^{\infty} \rho_{0i}\rho_{0j} \sup_{(t,x)\in\mathbb{R}^4} \psi_i\psi_j \int_{\mathbb{R}^3} dq \int_{\mathbb{R}^3} dq_1\, e^{-q^2-q_1^2}$$

$$\times \left| VW_i - VW_j + q\sqrt{\gamma_i} - q_1\sqrt{\gamma_j} + \gamma_i^{m_i}\left[\overline{\omega}_{0i}, x - \overline{u}_{0i}t\right] - \gamma_j^{m_j}\left[\overline{\omega}_{0j}, x - \overline{u}_{0j}t\right]\right|. \tag{3.24}$$

The limiting passage ($\beta_i \to +\infty$) in the expression for Δ' is equivalent to $\gamma_i \to +0$. The possibility of such a transition is ensured by the continuity of the expression (3.24) at the point $\gamma_i = 0$, by the condition of uniform convergence of series with common term of Theorem 3.1, i.e., one of the functions (3.2), and of the obvious estimate

$$|\gamma_i| \leqslant \frac{1}{\beta}$$

in accordance to Remark 3.2. In addition, we apply the lemma from [6] on the continuity of the supremum with respect to the parameter and the theorems on the continuity of integral and functional series with respect to the parameter.

For values of the exponent m_i greater than $\frac{1}{2}$, the low-temperature limit of the majorant Δ' after calculating the integral over the variable p coincides with equality (3.5). If m_i is equal to $\frac{1}{2}$, then an additional term appears under the sign of the supremum

$$2\left(p, \left[\overline{\omega}_{0i}, \widetilde{V}_i - \overline{u}_{0i}\right]\right)\psi_i(t,x).$$

Evaluating the additional term and calculating the integral over the variable p again, we obtain equality (3.7). In the case of values m_i from the interval $\left(\frac{1}{4}, \frac{1}{2}\right)$, we use the additional parallelism condition (3.8) and we get the limit value (3.5) again. For the boundary value $m_i = \frac{1}{4}$, we use the modification of the condition (3.3), namely, the average angular velocity in the form (3.9) that gives

$$\begin{aligned}
\Delta' = &\sum_{i=1}^{\infty} \rho_{0i}\pi^{-3/2}\int_{\mathbb{R}^3} dp\, e^{-p^2} \\
&\times \sup_{(t,x)\in\mathbb{R}^4}\left|\frac{\partial\psi_i}{\partial t} + \left(VW_i + s_i\gamma_i^{\frac{1}{4}}\left[\overline{\omega}_{0i}, x - \overline{u}_{0i}t\right] + p\sqrt{\gamma_i}, \frac{\partial\psi_i}{\partial x}\right)\right. \\
&\left. + 2\psi_i s_i^2\left(p, \overline{\omega}_{0i}\left(\overline{\omega}_{0i}, x\right) - \overline{\omega}_{0i}^2\left(x - \overline{u}_{0i}t\right)\right)\right| \\
&+ 2\sum_{\substack{i,j=1\\ i\neq j}}^{\infty}\frac{d^2\rho_{0i}\rho_{0j}}{\pi^2}\sup_{(t,x)\in\mathbb{R}^4}\psi_i\psi_j\int_{\mathbb{R}^3} dq\int_{\mathbb{R}^3} dq_1\, e^{-q^2-q_1^2} \\
&\times\left|VW_i - VW_j + q\sqrt{\gamma_i} - q_1\sqrt{\gamma_j} + \gamma_i^{\frac{1}{4}}s_i\left[\overline{\omega}_{0i}, x - \overline{u}_{0i}t\right] - \gamma_j^{\frac{1}{4}}s_j\left[\overline{\omega}_{0j}, x - \overline{u}_{0j}t\right]\right|.
\end{aligned} \tag{3.25}$$

Further, we pass to the limit in (3.25), as $\gamma_i \to +0$ and, in view of the estimate

$$\left|2\psi_i s_i^2\left(p, \overline{\omega}_{0i}\left(\overline{\omega}_{0i}, x\right) - \overline{\omega}_{0i}^2\left(x - \overline{u}_{0i}t\right)\right)\right| \leqslant 2\overline{\omega}_{0i}^2 s_i^2|p|\left(2|x| + |\overline{u}_{0i}|\,t\right),$$

we obtain equality (3.10), which exhausts the proof of all statements of the theorem. □

Let us formulate some sufficient conditions for minimizing the mixed error (2.12).

Corollary 3.3. *Let the function* $\psi_i(t,x)$ *in* (3.1) *have the form*

$$\psi_i(t,x) = C_i(x - VW_i t) \tag{3.26}$$

or

$$\psi_i(t,x) = D_i\left([x, VW_i]\right), \tag{3.27}$$

where the functions $C_i(\cdot)$ *and* $D_i(\cdot)$ *satisfy the conditions of Theorem* 3.1*.*

Let one of the following conditions be also true:

$$VW_i = VW_j, \tag{3.28}$$

$$\operatorname{supp}\psi_i \cap \operatorname{supp}\psi_j = \varnothing \quad (i \neq j), \tag{3.29}$$

$$d \to 0. \tag{3.30}$$

Then the following assertions hold:

(i) *If* $m_i > \frac{1}{2}$*, then the error* (2.12) *can be made arbitrarily small.*

(ii) *If* $\frac{1}{4} < m_i \leqslant \frac{1}{2}$ *and the condition* (3.8) *is fulfilled, then the error* (2.12) *is infinitesimally small.*

(iii) *For the boundary value* $m_i = \frac{1}{4}$*, we additionally require*

$$s_i \to +0, \tag{3.31}$$

such that the error (2.12) *can be made arbitrarily small.*

Proof. The first sum in low-temperature limit expression (3.5) turns to zero due to the form of arguments in (3.26), (3.27). Indeed, in case of (3.26), we have

$$\frac{\partial\psi_i}{\partial t} = -\left(VW_i, C_i'\right), \quad \frac{\partial\psi_i}{\partial x} = C_i',$$

while for the function $\psi_i(t,x)$ in (3.27), we obtain

$$\frac{\partial\psi_i}{\partial t} = 0, \quad \frac{\partial\psi_i}{\partial x} = \left[D_i', VW_i\right],$$

and hence the multiplier

$$\sup_{(t,x)\in\mathbb{R}^4} \left|\frac{\partial\psi_i}{\partial t} + \left(VW_i, \frac{\partial\psi_i}{\partial x}\right)\right|$$

tends to zero.

For values $m_i > \frac{1}{2}$, the second sum in (3.5) turns to zero if one of the conditions (3.28), (3.29) or (3.30) is fulfilled. If $m_i \in \left(\frac{1}{4}, \frac{1}{2}\right]$ and the condition of parallelism of vectors $\overline{\omega}_{0i}$ and $\left(\widetilde{V}_i - \overline{u}_{0i}\right)$ is fulfilled, then the mixed error (2.12) for the constructed distribution can be made arbitrarily small. For the boundary value

$m_i = \frac{1}{4}$, infinitesimal deviation (2.12) is achieved under the condition (3.31), which turns the following term to zero:

$$\frac{4}{\sqrt{\pi}} \sum_{i=1}^{\infty} \rho_{0i} s_i^2 \overline{\omega}_{0i}^2 \sup_{(t,x)\in\mathbb{R}^4} (2|x| + |\overline{u}_{0i}|\, t)\, \psi_i(t,x).$$

The statements of Corollary 3.3 are justified. □

Let us summarize: for all values of indicators $m_i \geqslant \frac{1}{4}$ from the representation of average angular velocity $\overline{\omega}_i$ according to (3.3), the distribution (2.10) with coefficient functions $\varphi_i(t,x)$ of the form (3.1) is an approximate solution to the Bryan–Pidduck equation (1.1)–(1.3) in the sense of mixed error (2.12) if we take into account equations (3.26), (3.27) and choose special hydrodynamic parameters of the distribution (2.1).

Consider another approach to constructing the solution of the Bryan–Pidduck equation without introducing an auxiliary function, namely, by abandoning the condition (3.1).

Theorem 3.4. *Let all functional series whose common term has the form (3.2) additionally multiplied by $e^{\beta_i \overline{\omega}_i^2 r_i^2}$ converge uniformly in time and space. Let the condition of parallelism of vectors (3.8) be preserved, and the condition (3.3) be fulfilled for $m_i \geqslant \frac{1}{2}$.*

Then there exists such a majorant Δ' that the estimate (3.4) holds and the following low-temperature limit holds:

$$\lim_{\beta_i\to+\infty} \Delta' = \sum_{i=1}^{\infty} \rho_{0i} \sup_{(t,x)\in\mathbb{R}^4} \left(\mu_i(t,x) \left| \frac{\partial\varphi_i}{\partial t} + \left(VW_i, \frac{\partial\varphi_i}{\partial x} \right) \right| \right)$$
$$+ 2\pi d^2 \sum_{\substack{i,j=1\\ i\neq j}}^{\infty} \rho_{0i}\rho_{0j} |VW_i - VW_j| \sup_{(t,x)\in\mathbb{R}^4} (\mu_i(t,x)\mu_j(t,x)\varphi_i\varphi_j), \quad (3.32)$$

where the function $\mu_i(t,x)$ is defined as follows:

$$\mu_i(t,x) = \begin{cases} e^{[\omega_{0i}, x - u_{0i}t]^2}, & m_i = \frac{1}{2}, \\ 1, & m_i > \frac{1}{2}. \end{cases} \quad (3.33)$$

Proof. In the obtained inequality (3.11), we write the form of Maxwellians (2.1) substituting the form of the flow density (2.2),

$$\int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega \left| D(f) - Q(f,f) \right| \leqslant \sum_{i=1}^{\infty} \rho_{0i} I^{3/2} \left(\frac{\beta_i}{\pi} \right)^3 e^{\beta_i \overline{\omega}_i^2 r_i^2}$$
$$\times \int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega e^{-\beta_i \left((V - \overline{V}_i)^2 + I\omega^2 \right)} \left| \frac{\partial\varphi_i}{\partial t} + \left(V, \frac{\partial\varphi_i}{\partial x} \right) \right|$$
$$+ 2 \sum_{\substack{i,j=1\\ i\neq j}}^{\infty} \frac{d^2 \rho_{0i}\rho_{0j}}{\pi^2} e^{\beta_i \overline{\omega}_i^2 r_i^2 + \beta_j \overline{\omega}_j^2 r_j^2} \varphi_i\varphi_j$$

$$\times \int_{\mathbb{R}^3} dq \int_{\mathbb{R}^3} dq_1 e^{-q^2-q_1^2} \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \overline{V}_i - \overline{V}_j \right|. \quad (3.34)$$

We calculate the integral on the right-hand side of inequality (3.34) over the space of angular velocities and make a substitution (3.18). As a result, we have the estimate

$$\begin{aligned} &\int_{\mathbb{R}^3} dV \int_{\mathbb{R}^3} d\omega \Big| D(f) - Q(f,f) \Big| \\ &\quad \leqslant \pi^{-3/2} \sum_{i=1}^{\infty} \rho_{0i} e^{\beta_i \overline{\omega}_i^2 r_i^2} \int_{\mathbb{R}^3} dp e^{-p^2} \left| \frac{\partial \varphi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + \overline{V}_i, \frac{\partial \varphi_i}{\partial x} \right) \right| \\ &\quad + 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{\infty} \frac{d^2 \rho_{0i} \rho_{0j}}{\pi^2} e^{\beta_i \overline{\omega}_i^2 r_i^2 + \beta_j \overline{\omega}_j^2 r_j^2} \varphi_i \varphi_j \\ &\quad \times \int_{\mathbb{R}^3} dq \int_{\mathbb{R}^3} dq_1 e^{-q^2-q_1^2} \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + \overline{V}_i - \overline{V}_j \right|. \end{aligned} \quad (3.35)$$

Let us move on to the supremum in terms of variables (t, x), the existence of which is provided by the imposed condition of Theorem 3.4 on uniform convergence of functional series with common term, where one of the functions (3.2) is additionally multiplied by the density exponent $e^{\beta_i \overline{\omega}_i^2 r_i^2}$. Using the representation of the angular velocity of the flow (3.3) and the condition (3.8), we indicate the form of the value of the majorant Δ':

$$\begin{aligned} \Delta' &= \pi^{-3/2} \sum_{i=1}^{\infty} \rho_{0i} \sup_{(t,x)\in\mathbb{R}^4} e^{\beta_i^{1-2m_i} [\overline{\omega}_{0i}, x - \overline{u}_{0i} t]^2} \int_{\mathbb{R}^3} dp\, e^{-p^2} \\ &\times \left| \frac{\partial \varphi_i}{\partial t} + \left(\frac{p}{\sqrt{\beta_i}} + VW_i + \beta_i^{-m_i} \left[\overline{\omega}_{0i}, x - \overline{u}_{0i} t \right], \frac{\partial \varphi_i}{\partial x} \right) \right| \\ &+ 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{\infty} \frac{d^2 \rho_{0i} \rho_{0j}}{\pi^2} \sup_{(t,x)\in\mathbb{R}^4} e^{\beta_i^{1-2m_i} [\overline{\omega}_{0i}, x - \overline{u}_{0i} t]^2 + \beta_j^{1-2m_j} [\overline{\omega}_{0j}, x - \overline{u}_{0j} t]^2} \\ &\times \varphi_i \varphi_j \int_{\mathbb{R}^3} dq \int_{\mathbb{R}^3} dq_1\, e^{-q^2-q_1^2} \\ &\times \left| \frac{q}{\sqrt{\beta_i}} - \frac{q_1}{\sqrt{\beta_j}} + VW_i - VW_j + \beta_i^{-m_i} \left[\overline{\omega}_{0i}, x - \overline{u}_{0i} t \right] - \beta_j^{-m_j} \left[\overline{\omega}_{0j}, x - \overline{u}_{0j} t \right] \right|. \end{aligned} \quad (3.36)$$

Next, we perform the limit transition $\beta_i \to +\infty$ arguing its possibility in the same way as in the proof of Theorem 3.1. After minor transformations, we get that the low-temperature limit of the majorant Δ' is given by (3.32) and (3.33), which completely exhausts the verification of the statement of the theorem. □

Remark 3.5. In order to achieve an infinitesimally small mixed error (2.12) for the constructed distribution (2.10) under condition that Δ' has a low-temperature limit (3.32), one can apply Corollary 3.3 with minor modification. As the coefficient functions $\varphi_i(t,x)$ we take the function (3.26) or (3.27). In this case, it is necessary that the condition (3.28) or (3.30) be satisfied, or the intersection of supports of the selected coefficient functions $\varphi_i(t,x)$ and $\varphi_j(t,x)$ be empty for different indices i and j.

A similar problem was considered earlier in [12] for the case of the hard sphere model, which is a physically simpler model of particles since, unlike rough spheres, it does not involve rotation of the molecule around its axis. The obtained results allow us to conclude that, in general, the result for hard spheres can be transferred to a more interesting and physically complex model of rough spheres.

4. Conclusions

In the paper, an approximate solution for the Bryan–Pidduck equation is constructed in the form of a countable modal linear combination of Maxwellian flows (2.10). As Maxwellians, a non-stationary, non-homogeneous Maxwellian is considered, which describes the eddy-like motion of gas or, in short, an eddy. Sufficient conditions are obtained under which some deviation between the parts of the Boltzmann equation can be made as arbitrarily small as desired.

From a physical point of view, the constructed solution describes, in some sense, the interaction of a countable number of Maxwellian flows. These flows rotate around some axes and move translationally. In addition, due to the imposed condition (3.3), the flows slow down the rotation in general simultaneously with cooling ($\beta_i \to +\infty$). The condition (3.30) can be interpreted as a consideration of the near-Knudsen gas flows.

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Взаємодія зліченного числа смерчоподібних течій для моделі Брайана–Піддака

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Ми розглядаємо нелінійне інтегро-диференціальне рівняння Больцмана для моделі шорсткуватих сфер. Побудовано наближений розв'язок у вигляді лінійної комбінації зліченної кількості максвеллівських мод з деякими коефіцієнтними функціями, що залежать від часу і просторової координати. Одержано достатні умови довільної мализни рівномірно інтегрального відхилу.

Ключові слова: рівняння Брайана–Піддака, шорсткуваті сфери, смерчоподібні течії, нескінченно-модальний розподіл