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Solutions of the Frobenius Coupled KP Equation

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In this paper, we firstly construct the coupled Schur function solution of the Frobenius coupled Kadomtsev–Petviashvili (KP) hierarchy as a generalization of the Schur function. The Frobenius coupled KP hierarchy contains a Frobenius coupled KP equation which has a potential application in the theory of two-layer shallow water waves. We further derive some regular Wronskian solution and non-Wronskian solutions of the Frobenius coupled KP equation.

Key words: Frobenius coupled KP hierarchy, Schur function, coupled Schur function solution, Wronskian solution, non-Wronskian solution.

Mathematical Subject Classification 2010: 34M55, 37K10.

1. Introduction

The Kadomtsev–Petviashvili (KP) hierarchy [1] is one of the most important integrable hierarchies and it arises in many different fields of mathematics and physics such as enumerative algebraic geometry, topological field and string theory. In [2], a new hierarchy called the Frobenius-valued Kadomtsev-Petviashvili hierarchy which takes values in a maximal commutative subalgebra of $gl(m, \mathbb{C})$ was constructed, meanwhile the relation between Frobenius manifold and dispersionless reduced Frobenius-valued KP hierarchy was discussed. For the same Frobenius version of Toda system, we consider the Hirota quadratic equation of the commutative version of extended multi-component Toda hierarchy in [3] which should be useful in Frobenius manifold theory. Because of logarithm terms, some extended Vertex operators are constructed in generalized Hirota bilinear equations which might be useful in topological field theory and Gromov-Witten theory. Later we defined a new multi-component BKP hierarchy which takes values in a Frobenius subalgebra of $gl(N, \mathbb{C})$ and also we gave the gauge transformations of this Frobenius BKP hierarchy [4]. Recently, in [5], we constructed the Frobenius-valued Sine-Gordon systems which generate some new coupled integrable equations. In [6], we constructed affine Weyl group symmetries of some newly-defined Frobenius Painlevé equations which can be derived from the Frobenius modified KP hierarchy. Then a natural question appears, "What about regular solutions, rational solutions and non-Wronskian solutions of the Frobenius KP equation?". In this paper, we will answer this question by considering a specific Frobenius KP equation called the Frobenius coupled KP equation.

This paper is arranged as follows. In Section 2, we recall some basic facts about the Frobenius coupled KP hierarchy. We further derive a regular Wronskian solution of the Frobenius coupled Kadomtsev–Petviashvili equation in Section 3. In Section 4, we further derive a non-Wronskian solution of the Frobenius coupled Kadomtsev–Petviashvili equation.

2. Frobenius coupled KP hierarchy

In this section, we will use the factorization problem to derive the Lax equations of Frobenius coupled KP hierarchy [2]. Here we will consider the linear space of the complex 2×2 matrix-valued function $g : \mathbb{R} \to M_2(\mathbb{C})$ with the derivative operator ∂. Then the set $\mathfrak{g}$ of Laurent series in ∂ as an associative algebra is a Lie algebra under the standard commutator. This Lie algebra has the following important splitting:

$$\mathfrak{g} = \mathfrak{g}_+ \oplus \mathfrak{g}_-, \tag{2.1}$$

where

$$\mathfrak{g}_+ = \Big\{ \sum_{j\geq 0} C_j(x)\partial^j,\ C_j(x) \in M_2(\mathbb{C}) \Big\},\quad \mathfrak{g}_- = \Big\{ \sum_{j<0} C_j(x)\partial^j,\ C_j(x) \in M_2(\mathbb{C}) \Big\}.$$

The splitting (2.1) leads us to consider the factorization of $g \in G$:

$$g = g_-^{-1} \circ g_+, \quad g_\pm \in G_\pm, \tag{2.2}$$

where $G_\pm$ have $\mathfrak{g}_\pm$ as their Lie algebras. G_+ is the set of invertible linear operators of the form $\sum_{j\geq 0} g_j(x)\partial^j$, while G_- is the set of invertible linear operators of the form $1 + \sum_{j<0} g_j(x)\partial^j$. This algebra has a Frobenius subalgebra $F_2 = \mathbb{C}[\Gamma]/(\Gamma^2)$ and $\Gamma = (\delta_{i,j+1})_{ij} \in gl(2, \mathbb{C})$. Denote $F_2(\partial) := \mathfrak{g}_F$, then we have the splitting

$$\mathfrak{g}_F = \mathfrak{g}_{F+} \oplus \mathfrak{g}_{F-}, \tag{2.3}$$

where

$$\mathfrak{g}_{F+} = \Big\{ \sum_{j\geq 0} X_j(x)\partial^j,\ X_j(x) \in F_2 \Big\},\quad \mathfrak{g}_{F-} = \Big\{ \sum_{j<0} X_j(x)\partial^j,\ X_j(x) \in F_2 \Big\}.$$

The Lax operator of the Frobenius coupled KP hierarchy is as

$$L = \partial + \sum_{i\geq 1} u_i \partial^{-i}, \tag{2.4}$$

where u_i takes values in the Frobenius subalgebra F_2, $u_1 = uE_2 + v\Gamma$.

The Frobenius coupled KP hierarchy is defined by the following Lax equations:

$$\partial_k L = [(B_k)_+, L], \quad B_k = L^k, \quad k \geq 1. \tag{2.5}$$

One can write the operators L in a dressing form as

$$L = \Phi \circ \partial \circ \Phi^{-1}, \tag{2.6}$$

where

$$\Phi = E_2 + \sum_{i\geq 1} a_i \partial^{-i} = \begin{pmatrix} 1 + \sum_{i\geq 1} a_{i0}\partial^{-i} & 0 \\ \sum_{i\geq 1} a_{i1}\partial^{-i} & 1 + \sum_{i\geq 1} a_{i0}\partial^{-i} \end{pmatrix}. \tag{2.7}$$

Here E_2 means the 2×2 unit matrix and

$$u = -a_{10x}, \quad v = -a_{11x}.$$

Given L, the dressing operators Φ are determined uniquely up to a multiplication to the right by operators with constant coefficients. The dressing operator Φ takes values in a Frobenius–Volterra group in G_-. The Frobenius coupled KP hierarchy (2.5) can be redefined as

$$\frac{\partial \Phi}{\partial t_k} = -(L^k)_- \circ \Phi,$$

with $k \geq 1$. In the Frobenius coupled KP hierarchy, we can derive an equation

$$(4u_t - 12uu_x - u_{xxx})_x - 3u_{yy} = 0, \quad (4v_t - 12uv_x - 12u_x v - v_{xxx})_x - 3v_{yy} = 0,$$

which is the Frobenius coupled KP equation [2]. As we know, Lax pair is equivalent to the bilinear equation, for a tau function $\sigma = \sigma(x)$, $\tau = \tau(x)$. The symbol $[k]$ denotes $(k, k^2/2, k^3/3, \ldots)$. Now we give the definition of the tau functions (τ, σ):

$$1 + \sum_{i\geq 1} a_{i0} k^{-i} = \frac{\tau(\boldsymbol{x} - [k^{-1}])}{\tau(\boldsymbol{x})},$$

$$\sum_{i\geq 1} a_{i1} k^{-i} = \frac{\tau(\boldsymbol{x})\sigma(\boldsymbol{x} - [k^{-1}]) - \tau(\boldsymbol{x} - [k^{-1}])\sigma(\boldsymbol{x})}{\tau^2(\boldsymbol{x})},$$

where

$$\boldsymbol{x} := (x_1, x_2, \cdots).$$

We can further get

$$a_{10} = -\frac{\partial_x \tau(\boldsymbol{x})}{\tau(\boldsymbol{x})},$$

$$a_{11} = \frac{\partial_x \tau(\boldsymbol{x})\sigma(\boldsymbol{x}) - \tau(\boldsymbol{x})\partial_x \sigma(\boldsymbol{x})}{\tau^2(\boldsymbol{x})}.$$

The Frobenius coupled KP hierarchy can be redefined by the bilinear equation

$$\begin{cases} \sum_{i+j=-2} Z_i^- \tau \otimes Z_j^+ \tau = 0, \\ \sum_{i+j=-2} Z_i^- \tau \otimes Z_j^+ \sigma + \sum_{i+j=-2} Z_i^- \sigma \otimes Z_j^+ \tau = 0. \end{cases} \tag{2.8}$$

And the operator $Z_i^{\pm}$ in (2.8) comes from the vertex operators

$$Z^{\pm}(k) = \sum_{n \in Z} Z_n^{\pm} k^n = e^{\pm \varphi(\boldsymbol{x},k)} e^{\mp \varphi(\bar{\partial}_{\boldsymbol{x}}, k^{-1})},$$

where

$$\varphi(\boldsymbol{x}, k) = \sum_{n=1}^{\infty} x_n k^n, \quad \bar{\partial}_{\boldsymbol{x}} = (\frac{\partial}{\partial x_1}, \frac{\partial}{2\partial x_2}, \cdots).$$

3. Coupled Schur function solutions of Frobenius coupled KP hierarchy

As we know, for a partition $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_l)$, as a sequence of non-negative integers such that $\lambda_1 \geq \lambda_2 \geq \cdots \geq 0$. The number $l = l(\lambda) = \{i \mid \lambda_i \neq 0\}$ is called the length of λ, and the sum $|\lambda| = \lambda_1 + \lambda_2 + \cdots + \lambda_l$ is called the weight of λ. As we know, the tau function of the KP hierarchy can be expressed by the Schur function

$$S_\lambda(\boldsymbol{x}) = \det\big(p_{\lambda_i - i + j}(\boldsymbol{x})\big)_{1 \leq i,j \leq l},$$

where p_n ($n \in \mathbb{Z}$) is defined by the generating function

$$\sum_{n \in \mathbb{Z}} p_n(\boldsymbol{x}) k^n = e^{\varphi(\boldsymbol{x},k)}, \tag{3.1}$$

i.e., $p_n = 0$ ($n < 0$), $p_0 = 1$, and

$$p_n = \sum_{k_1 + 2k_2 + \cdots + nk_n = n} \frac{{x_1}^{k_1} {x_2}^{k_2} \cdots {x_n}^{k_n}}{k_1! k_2! \cdots k_n!}.$$

If we count the degree of the variable x_n as $\deg x_n = n$, then S_λ is a weighted homogeneous polynomial of degree $|\lambda|$.

For the Frobenius coupled KP hierarchy, two tau functions are needed. The two tau functions (τ, σ) correspond to a pair of partitions λ and $\hat{\lambda}$. Note that the coupled Schur function $S_{[\lambda;\hat{\lambda}]}$ is a special case of the F_2-valued universal character,

$$S_{[\lambda;\hat{\lambda}]}(\boldsymbol{x}) = \det\big(p_{\lambda_i - i + j}(\boldsymbol{x}) E + p_{\hat{\lambda}_i - i + j}(\boldsymbol{x}) \Gamma\big),$$

which can be the tau function $\tau E_2 + \sigma \Gamma$ of the Frobenius KP hierarchy. It means that the specific tau functions of the Frobenius KP hierarchy are:

$$\tau = S_\lambda(\boldsymbol{x}) = \det\big(p_{\lambda_i - i + j}(\boldsymbol{x})\big)_{1 \leq i,j \leq l},$$

$$\sigma = \sum_{k=1}^{l} S_{(\lambda_1,\cdots,\hat{\lambda}_k,\cdots,\lambda_l)}(\boldsymbol{x}).$$

By means of these operators, the coupled Schur function solutions of the Frobenius KP hierarchy can be expressed using the vertex operators

$$\tau(\boldsymbol{x}) = Z^+_{\lambda_1} \dots Z^+_{\lambda_l} 1, \tag{3.2}$$

$$\sigma = \sum_{k=1}^{l} Z^+_{\lambda_1} \dots Z^+_{\hat{\lambda}_k} \dots Z^+_{\lambda_l} 1. \tag{3.3}$$

4. Regular Wronskian solution of the Frobenius coupled KP equation

The Frobenius coupled KP equation is the following partial differential equation in $2+1$ dimensions:

$$(4u_t - 12uu_x - u_{xxx})_x - 3u_{yy} = 0, \tag{4.1}$$

$$(4v_t - 12uv_x - 12u_x v - v_{xxx})_x - 3v_{yy} = 0. \tag{4.2}$$

It is known that the solutions of the Frobenius coupled KP equation can be expressed in terms of the τ-function:

$$u(x,y,t) = \frac{\partial^2}{\partial x^2} \log \tau(x,y,t),$$

$$v(x,y,t) = \frac{\partial^2}{\partial x^2}\left(\frac{\sigma(x,y,t)}{\tau(x,y,t)}\right) = \left(\frac{\tau(\boldsymbol{x})\partial_x\sigma(\boldsymbol{x}) - \partial_x\tau(\boldsymbol{x})\sigma(\boldsymbol{x})}{\tau^2(\boldsymbol{x})}\right)_x.$$

Consider a class of solutions whose τ-function is given by the Wronskian determinant

$$\tau(x,y,t) = Wr(f_1,\dots,f_N) = \det\begin{pmatrix} f_1 & f_2 & \cdots & f_N \\ f_1' & f_2' & \cdots & f_N' \\ \vdots & \vdots & & \vdots \\ f_N^{(N-1)} & f_2^{(N-1)} & \cdots & f_N^{(N-1)} \end{pmatrix}, \tag{4.3}$$

$$\sigma(x,y,t) = \sum_{i=1}^{N} Wr(f_1,\dots,g_i,\dots,f_N), \tag{4.4}$$

with $f^{(i)} = \partial^i f/\partial x^i$, and where the functions $\{f_i, g_i\}_{i=1}^N$ are a set of linearly independent solutions of the linear system:

$$\frac{\partial f_i}{\partial y} = \frac{\partial^2 f_i}{\partial x^2}, \qquad \frac{\partial f_i}{\partial t} = \frac{\partial^3 f_i}{\partial x^3},$$

$$\frac{\partial g_i}{\partial y} = \frac{\partial^2 g_i}{\partial x^2}, \qquad \frac{\partial g_i}{\partial t} = \frac{\partial^3 g_i}{\partial x^3}.$$

This Wronskian form includes the line-soliton solutions of the Frobenius coupled KP equation, which are real non-singular solutions localized along certain directions in the (x, y)-plane and decay exponentially everywhere else.

Let $\mathrm{Gr}(N, M)$ be the set of all N-dimensional subspace of $\mathbb{R}^M$. Let $\{E_j : j = 1, \cdots, M\}$ be a basis of $\mathbb{R}^M$, i.e., $\mathrm{Span}_{\mathbb{R}}\{E_j : j = 1, \dots, M\} = \mathbb{R}^M$. Let $\{f_i : i = 1, \dots, N\}$, $N \le M$ be a basis of an N-dimensional subspace

$$f_i = \sum_{j=1}^{M} a_{ij} E_j, \quad i = 1, \dots, N,$$

$A = (a_{ij}) \in M_{N\times M}(\mathbb{R})$ ={the set of $N \times M$ matrices of rank N}. The matrix A identifies a point on $\mathrm{Gr}(N, M)$ and has NM parameters

$$\mathrm{Gr}(N, M) \backsimeq \mathrm{GL}_N(\mathbb{R})\backslash M_{N\times M}(\mathbb{R})$$

and $\dim \mathrm{Gr}(N, M) = NM - N^2 = N(M - N)$.

To see the Wroskian solutions clearly, we take some examples.

Example 4.1. Suppose a one-soliton solution is obtained by choosing $N = 1$ in equation (4.3), and $\tau(x, y, t) = e^{\theta_1} + e^{\theta_2}$, where

$$\theta_m(x, y, t) = k_m x + k_m^2 y + k_m^3 t + \theta_{m,0},$$

$\sigma(x, y, t) = e^{\vartheta_1} + e^{\vartheta_2}$, where

$$\vartheta_m(x, y, t) = \lambda_m x + \lambda_m^2 y + \lambda_m^3 t + \vartheta_{m,0}$$

with $\theta_{m,0}, \vartheta_{m,0}, k_m, \lambda_m$ for $m = 1, 2$ being constants, and $k_1 < k_2$. The above choices yield the travelling-wave solution

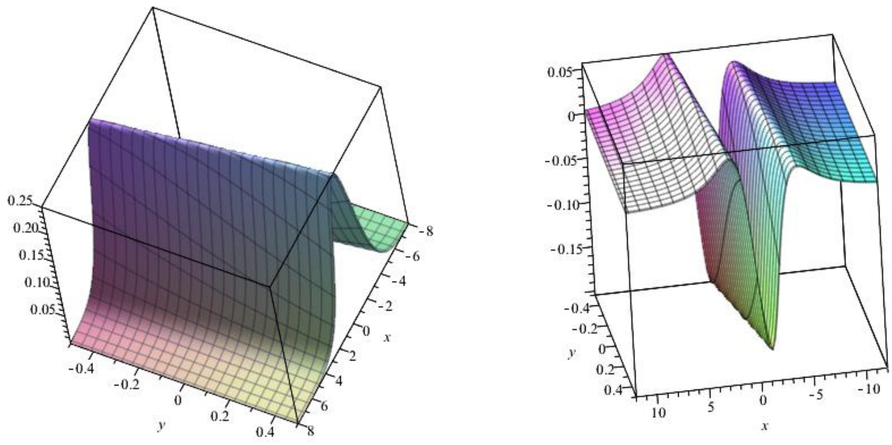
$$\begin{aligned} u(x, y, t) &= \frac{1}{2}(k_2 - k_1)^2 \operatorname{sech}^2 \frac{1}{2}(\theta_2 - \theta_1), \\ v(x, y, t) &= \left(\frac{(e^{\theta_1} + e^{\theta_2})(\lambda_1 e^{\vartheta_1} + \lambda_2 e^{\vartheta_2}) - (k_1 e^{\theta_1} + k_2 e^{\theta_2})(e^{\vartheta_1} + e^{\vartheta_2})}{(e^{\theta_1} + e^{\theta_2})^2}\right)_x . \end{aligned}$$

The graph of the one-soliton solution (u, v) can be seen in Fig. 4.1.

The solitary wave given by equation (4.7) is localized in the (x, y)-plane along the line $L : \theta_1 = \theta_2$ whose normal has the slope $c = k_1 + k_2$. The one-soliton solution is characterized by two physical parameters, namely, the *soliton amplitude* $a = k_2 - k_1$ and the *soliton direction* $c = k_1 + k_2 = \tan\alpha$, where α is the angle, measured counterclockwise, between the line and the positive y-axis. When $c = 0$ (equivalently, $k_1 = -k_2$), the solution in equation (4.7) becomes y-independent and reduces to the one-soliton solution of the following Frobenius coupled Korteweg-de Vries (KdV) equation:

$$4u_t - 12uu_x - u_{xxx} = 0, \tag{4.5}$$

$$4v_t - 12uv_x - 12u_x v - v_{xxx} = 0. \tag{4.6}$$



(a) The function u. (b) The function v.

Fig. 4.1: The one-soliton solution (u, v) for $k_1 = 1.5$, $k_2 = 0.5$, $\lambda_1 = 1.2$, $\lambda_2 = 0.7$ of the Frobenius coupled KP equation.

Example 4.2. Suppose a (1,2)-soliton solution is obtained by choosing $N = 1$ in equation (4.3) above, and $\tau(x, y, t) = e^{\theta_1} + e^{\theta_2} + e^{\theta_3}$, where

$$\theta_m(x, y, t) = k_m x + k_m^2 y + k_m^3 t + \theta_{m,0},$$

$\sigma(x, y, t) = e^{\vartheta_1} + e^{\vartheta_2} + e^{\vartheta_3}$, where

$$\vartheta_m(x, y, t) = \lambda_m x + \lambda_m^2 y + \lambda_m^3 t + \vartheta_{m,0},$$

with $\theta_{m,0}, \vartheta_{m,0}, k_m, \lambda_m$ for $m = 1, 2, 3$ being constants, and $k_1 < k_2 < k_3$. The above choices yield the travelling-wave solution

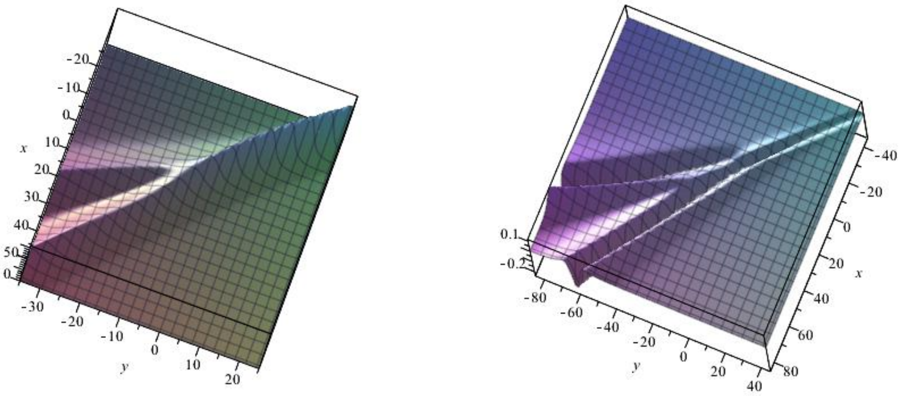
$$\begin{aligned} u(x, y, t) &= \left(\frac{k_1 e^{\theta_1} + k_2 e^{\theta_2} + k_3 e^{\theta_3}}{e^{\theta_1} + e^{\theta_2} + e^{\theta_3}} \right)_x, \\ v(x, y, t) &= \left(\frac{(e^{\theta_1} + e^{\theta_2} + e^{\theta_3})(\lambda_1 e^{\vartheta_1} + \lambda_2 e^{\vartheta_2} + \lambda_3 e^{\vartheta_3})}{(e^{\theta_1} + e^{\theta_2} + e^{\theta_3})^2} \right. \\ &\quad \left. - \frac{(k_1 e^{\theta_1} + k_2 e^{\theta_2} + k_3 e^{\theta_3})(e^{\vartheta_1} + e^{\vartheta_2} + e^{\vartheta_3})}{(e^{\theta_1} + e^{\theta_2} + e^{\theta_3})^2} \right)_x. \end{aligned} \tag{4.7}$$

The graph of the (1,2)-soliton solution (u, v) can be seen in Fig. 4.2.

5. Non-Wronskian solution of the Frobenius coupled KP equation

A transformation $u = \alpha_x|_{x \to -\frac{x}{4}}$, $v = \beta_x|_{x \to -\frac{x}{4}}$ can change the Frobenius coupled KP equation (4.1) into the potential Frobenius coupled KP equation:

$$\begin{aligned} (\alpha_t + 6\alpha_x^2 + \alpha_{xxx})_x + 3\alpha_{yy} &= 0, \\ (\beta_t + 12\beta_x \alpha_x - \beta_{xxx})_x + 3\beta_{yy} &= 0. \end{aligned}$$



(a) The function u. (b) The function v.

Fig. 4.2: The (1,2)-soliton solution (u, v) for $k_1 = 0.1$, $k_2 = 0.5$, $k_3 = 0.9$, $\lambda_1 = 0.2$, $\lambda_2 = 0.6$, $\lambda_3 = 0.8$ of the Frobenius coupled KP equation.

The above potential Frobenius coupled KP equation can have the following Non-Wronskian solution in the theorem below.

Theorem 5.1. *The potential Frobenius coupled KP equation can have the solution*

$$\alpha = \partial_x(\log A),$$
$$\beta = \partial_x\left(\frac{B}{A}\right),$$

where

$$A = \det(a_{rs}), \qquad B = \sum_j \det(a_{rs})|_{a_{*j}\to b_{*j}},$$
$$a_{rs} = \delta_{rs}e_{r1} + \frac{1}{a_r + c_s}, \qquad b_{rs} = \delta_{rs}e_{r2} - \frac{b_r + d_s}{(a_r + c_s)^2},$$
$$e_{r1} = \exp(-(a_r + c_r)x + (a_r^2 - c_r^2)y - (a_r^3 + c_r^3)t),$$
$$e_{r2} = [-(b_r + d_r)x + 2(a_r b_r - c_r d_r)y - \frac{1}{2}(a_r^2 b_r + c_r^2 d_r)t]e_{r1}.$$

Proof. The prove of the theorem is standard and similar to that from [7]. Just after doing the transformation

$$b_r \to a_r + b_r\Gamma,\ c_r \to c_r + d_r\Gamma,$$

the whole proof of Theorem 6.16 in [7] will lead to the results of this theorem. □

To see the above theorem clearly, we choose the values of $a_r, b_r,\ c_r, d_r$ to be constants, i.e., $a_r = a, b_r = b,\ c_r = c, d_r = d$; for $r = 1, 2, \cdots, n$. The potential

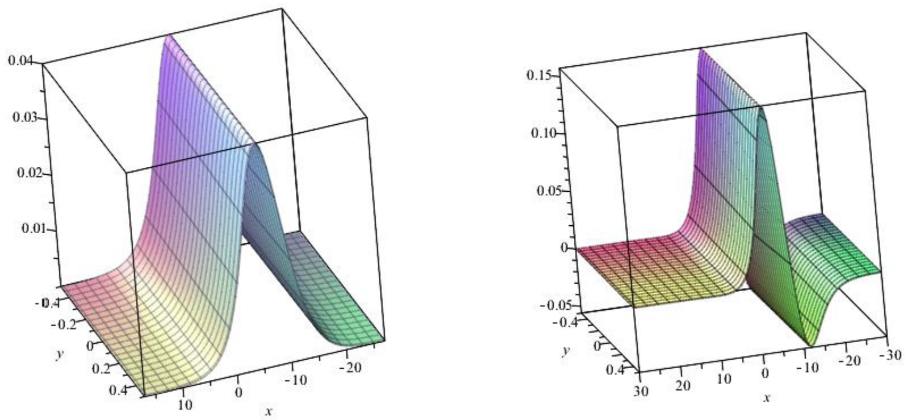
Frobenius coupled KP equation can have the solution

$$\alpha = \frac{n}{e^{h_1} + n\gamma}, \qquad \beta = -\frac{n(h_2 e^{h_1} + n\eta)}{(e^{h_1} + n\gamma)^2},$$

where

$$\begin{aligned} h_1 &= -(a+c)x + (a^2 - c^2)y + 4(a^3 + c^3)t, \\ h_2 &= -(b+d)x + 2(ab - cd)y + 2(a^2 b + c^2 d)t, \\ \gamma &= \frac{1}{a+c}, \qquad \eta = -\frac{b+d}{(a+c)^2}. \end{aligned}$$

Then the non-Wronskian solution (u, v) can be seen in Fig. 5.1.



(a) The function u. (b) The function v.

Fig. 5.1: The one-soliton solution (u, v) for $a = 0.1$, $b = 0.2$, $c = 0.3$, $d = 0.4$, $n = 5$ of the Frobenius coupled KP equation.

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Розв'язки пов'язаного за Фробеніусом рівняння КП

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У цій статті ми спочатку конструюємо пов'язану функцію Шура, що розв'язує алгеброзначну за Фробеніусом ієрархію Кадомцева–Петвіашвілі (КП), і є узагальненням функції Шура. Пов'язана за Фробеніусом ієрархія КП містить у собі пов'язане за Фробеніусом рівняння КП, що має можливі застосування у теорії двошарових мілководних хвиль. Далі ми отримуємо деякий регулярний вронскіанний розв'язок, а також невронскіанні розв'язки пов'язаного рівняння КП.

Ключові слова: пов'язана за Фробеніусом ієрархія КП, функція Шура, розв'язок типу пов'язаної функції Шура, вронскіанний розв'язок, невронскіанний розв'язок.